

## Progression in teaching addition facts

### Group A: Year 1

1. Adding 1 (e.g.  $7 + 1$  and  $1 + 7$ )
2. Doubles of numbers to 5 (e.g.  $4 + 4$ )
3. Adding 2 (e.g.  $4 + 2$  and  $2 + 4$ )
4. Number bonds to 10 (e.g.  $8 + 2$  and  $2 + 8$ )
5. Adding 10 to a number (e.g.  $5 + 10$  and  $10 + 5$ )
6. Adding 0 to a number (e.g.  $3 + 0$  and  $0 + 3$ )
7. The ones without a family!  $5 + 3$ ,  $3 + 5$ ,  $6 + 3$ ,  $3 + 6$

Knowing these facts by the end of Year 1 will mean children will know 87 of the 121 addition facts in the grid.

### Group B: Year 2

Children have 34 addition facts left to learn - they are the ones which bridge 10. While a few adults have instant recall of all of these, most rely on strategies for some. Our aim for children is that they use known facts or derived fact strategies to quickly recall or derive each fact. We need to ensure that all children move beyond counting based strategies. This will require careful teaching of the strategies combined with plenty of practice.

8. Doubles of numbers to 10 (e.g.  $7 + 7$ )
9. Near doubles (e.g.  $5 + 6$  and  $6 + 5$ )
10. Bridging (e.g.  $8 + 4$  and  $4 + 8$ )
11. Compensating

} Note that these 3 strategies can often be used interchangeably, e.g. for  $8 + 9$ , some people will use near doubles (e.g.  $8 + 8 + 1$ ), some will use bridging (e.g.  $8 + 2 + 7$ ) and some will use compensating ( $8 + 10 - 1$ )

**N.B. Before the children are ready to learn bridging as a strategy, they need to be able to partition all single digit numbers.** See separate doc for an approach for this.

For each of these 11 steps, a suggested teaching approach is laid out below, including manipulatives/images, key teaching points and a suggested teaching progression.

## Step 1: Adding 1 to a number

### Images/manipulatives

A numbered number line



Numicon pieces



### Key teaching points

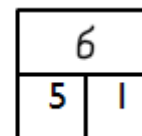
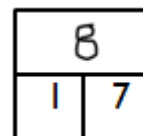
**1<sup>st</sup> key point:** Adding 1 to a number is the same as '1 more than' that number

**2<sup>nd</sup> key point: Commutativity**  $7 + 1 = 1 + 7$

### Teaching progression:

**Concrete:** Use equipment and a numbered number line to be able to say what is 1 more than any number to 10.

**Pictorial:** Represent this knowledge in part-part-whole diagrams



**Abstract:** Record this knowledge using number sentences;

Model that these can be expressed **commutatively**  $1 + 7 = 8$  or  $7 + 1 = 8$

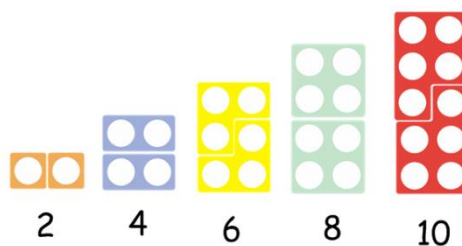
Model that these can also be expressed as **partitioning** the whole

$$8 = 1 + 7 \text{ or } 8 = 7 + 1$$

## Step 2: Doubles of numbers to 5

### Images/manipulatives

Numicon pieces



Doubles written up

$$1 + 1 = 2$$

$$2 + 2 = 4$$

$$3 + 3 = 6$$

$$4 + 4 = 8$$

$$5 + 5 = 10$$

### Key teaching points

**1<sup>st</sup> key point:** Our doubles of numbers to 5 are all even numbers [as appropriate you can lead children to the idea that doubling a whole number always gives us an even number]

**2<sup>nd</sup> key point:** We need to learn our doubles off by heart!

### Teaching progression:

**Awareness of odd and even:** Be able to identify numbers as odd or even, using Numicon as a visual image

**Fluency in odds and evens counting:** Practice counting in even numbers

**Understanding of what doubles is:** “Double 5” = “Two lots of 5” [spoken] =  $5 + 5$ . Can the children show you these with Numicon pieces or fingers on each hand?

**Noticing patterns:** Look as a class at the doubles pattern and relate to even numbers

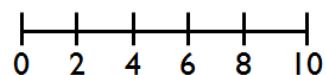
**PRACTICE:** Now you need to play LOTS of doubles games until the children all know their doubles of numbers to 5 off by heart. This is one of the sets which the children just need to memorise.

Represent in part part whole models and in number sentences.

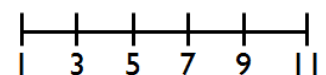
### Step 3: Adding 2 to a number

#### Images/manipulatives

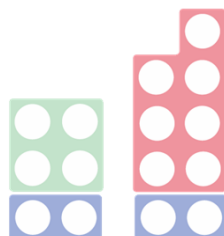
An evens numberline etc



An odds numberline etc



Numicon pieces



#### Key teaching points

**1<sup>st</sup> key point:** When we add 2 to a number, we are working within our odds and evens counting pattern

**2<sup>nd</sup> key point: Commutativity**  $7 + 2 = 2 + 7$

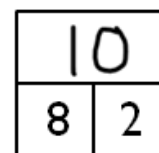
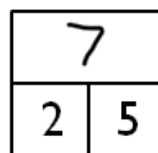
#### Possible teaching progression:

**Awareness of odd and even:** Be able to identify numbers as odd or even, using Numicon as a visual image

**Fluency in odds and evens counting:** Practice counting in odds and evens to 20, forwards and backwards until fluent. Use odd and even number lines for support.

**Concrete:** Use Numicon to see that when we add 2 to a number (or when we add a number to 2) we are just making the next odd/even number.

**Pictorial:** Represent this knowledge in part-part-whole diagrams

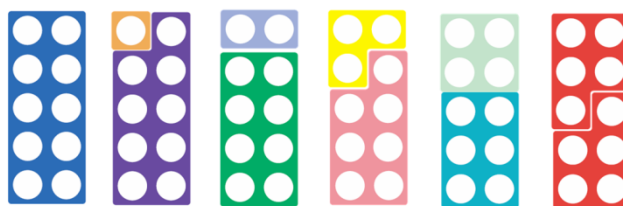


**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

## Step 4: Number bonds to 10

### Images/manipulatives

Numicon pieces



Number bonds to 10 written up

$$0 + 10 = 10$$

$$10 + 0 = 10$$

$$1 + 9 = 10$$

$$9 + 1 = 10$$

$$2 + 8 = 10$$

$$8 + 2 = 10$$

$$3 + 7 = 10$$

$$7 + 3 = 10$$

$$4 + 6 = 10$$

$$6 + 4 = 10$$

$$5 + 5 = 10$$

### Key teaching points

**1<sup>st</sup> key point:** Our number bonds to 10 are always odd + odd OR even + even

**2<sup>nd</sup> key point: Commutativity**  $6 + 4 = 4 + 6$

**3<sup>rd</sup> key point:** We need to learn our number bonds to 10 off by heart!

### Teaching progression:

**Awareness of odd and even:** Be able to identify numbers as odd or even, using Numicon as a visual image

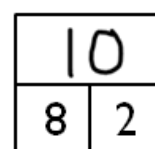
**Exploring different ways of making up 10:** Using the Numicon for support, notice that the number bonds to 10 are always odd + odd or even + even

**PRACTICE:** Now you need to play LOTS of doubles games until the children all know their doubles of numbers to 5 off by heart. This is one of the sets which the children just need to memorise.

Represent in part part whole models and in number sentences.

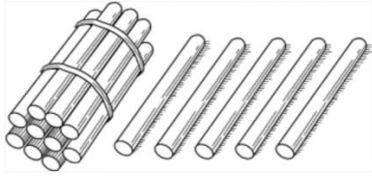
**Pictorial:** Represent this knowledge in part-part-whole diagrams

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

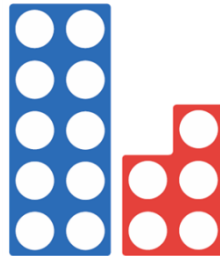


## Step 5: Adding 10 to a number

### Images/manipulatives



Base ten, e.g. straws, Numicon



### Key teaching points

**1<sup>st</sup> key point:** When we add 10 to a number we can use our place value knowledge to combine the numbers

**2<sup>nd</sup> key point: Commutativity**  $10 + 5 = 5 + 10$

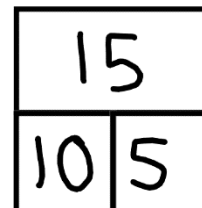
### Teaching progression:

**Place value experience:** Make up 'teens' numbers with place value equipment e.g. straws (or Numicon/Dienes).

**Relate place value representation to notation:** *"This is the number fifteen. We write it 15 because there is one ten and five ones."*

**Pictorial:** Represent this knowledge in part-part-whole diagrams

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole



## Step 6: Adding 0 to a number

### Images/manipulatives

Counters/straws/Numicon would all do here.

### Key teaching points

**1<sup>st</sup> key point:** When we add 0 to a number we are adding nothing, and so our starting number remains the same. [Misconception here is that  $7 + 0 = 0$ ]

**2<sup>nd</sup> key point: Commutativity**     $0 + 4 = 4 + 0$

### Teaching progression:

**Practical experience of making up number sentences involving 0:** Show me 0. Now add 4. How much do you have? Show me 4. Now add 0. How much do you have?

**Stem sentence:** *“When we add 0, we don’t change the quantity.”*

**Pictorial:** Represent this knowledge in part-part-whole diagrams

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

### **Step 7: The ones without a family**

The only remaining Y1 facts are  $6 + 3$  &  $3 + 6$  and  $3 + 5$  &  $5 + 3$ . These just need to be learnt. Fluent children often relate  $6 + 3$  to the counting in 3s pattern.

For  $5 + 3$  and  $3 + 5$  (indeed for any addition fact involving 5) I have found that teaching children to recognise the standard “finger pattern” for 8 of 5 fingers and 3 fingers is fairly easy, then this can be related to  $5 + 3$  and  $3 + 5$  (incidentally it is worth getting all reception children to recognise 6, 7, 8, and 9 when presented in this way, then they already ‘know’  $5 + 1$ ,  $5 + 2$ , and  $5 + 4$  as well (they just need to be *taught* that they already know them!).



## Step 8: Double 6, 7, 8 and 9

### Images/manipulatives



Double sided counters can model double 6, 7, 8 and 9 as double “5 and a bit” (i.e. double 8 is double 5 add double 3)



Numicon will allow the children to see that doubles of whole numbers are always even numbers

### Key teaching points

**1<sup>st</sup> key point:** Doubles of whole numbers are always even.

**2<sup>nd</sup> key point:** We need to learn our doubles off by heart!

### Teaching progression:

**[From Y1, children should be able to identify even numbers and know that a double means two lots of]**

Teach as follows:

**Double 6:** use the clock face. 6 at the bottom, 12 at the top.

**Double 7:** explain that two weeks is called a fortnight because it has 14 nights. There are 7 days in a week, so double 7 is 14.

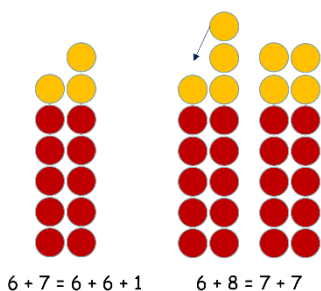
**Double 8 & double 9:** for a few children, remembering which is 16 and which is 18 seems particularly hard. There is no substitute for practice, asking any target children this many times each day for a week, and knowing which children don't yet know it.

**Relating to inverse.** What is half of 14 etc.

Once the facts are learnt, represent in PPW and number sentences as before.

## Step 9: Near doubles

### Images/manipulatives



Adjacent numbers well recognised as being 'near doubles' but 'one up one down' (i.e. second model shown here) is also a really nice efficient use of doubling.

### Key teaching points

**1<sup>st</sup> key point:** I can add adjacent numbers by doing 'double and 1 more'

**2<sup>nd</sup> key point:** I can add number with a difference of 2 (e.g.  $6 + 8$ ) by doubling the number in between them (i.e. by doubling 7 in this case)

**3<sup>rd</sup> key point:** Commutativity:  $5 + 6 = 6 + 5$

### Teaching progression:

**Fluency in doubles:** Will already have been secured

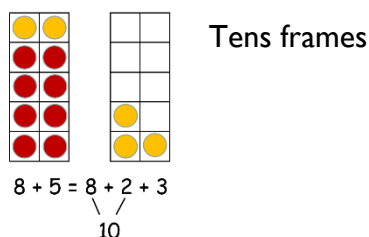
**Adjacent numbers:** Will be double the smaller number, add 1. OR double the larger number, subtract 1.

**Difference of 2:**  $5 + 7$ ,  $6 + 8$ ,  $7 + 9$

Then PPW and practice with number sentences as before

## Step 10: Bridging

### Images/manipulatives



### Key teaching points

**1<sup>st</sup> key point:** Bridging through ten can help us to calculate additions with a 'teens' total

**2<sup>nd</sup> key point:** Commutativity:  $5 + 8 = 8 + 5$

### Teaching progression:

**Partitioning single digit numbers:** Children **HAVE** to be able to do this to bridge. Calculating e.g.  $8 + 5$  by bridging requires partitioning the 5 into 2 and 3

**What makes ten?:** Children need to be able to make ten from 7, 8 and 9 (which are most likely to be involved in bridging facts)

**Tens frames (concrete):** Make up the two quantities with counters on adjacent tens frames, then rearrange as shown above.

**Symbolic:** Practice recording as number sentences (as shown above)

**PPW:** Move to filling in PPW as shown here:

8	5

**Number sentences (Abstract):** Children in the end should be able to solve  $8 + 5$  (etc) presented as number sentences by thinking "8 + 2 + 3" in their heads

**Comparison to other strategies:** Highlight that we can also use e.g. near doubles to solve some bridging facts (e.g.  $8 + 7$ )

## Step 11: Compensating/adjusting

### Images/manipulatives

Probably won't need to a manipulative for this (the children will already be fluent in

e.g.  $5 + 10$  and  $10 + 5$ )

$$5 + 10 = 15 \quad \text{so}$$

$$5 + 9 = 14$$

### Key teaching points

**1<sup>st</sup> key point:** By subtracting one from 'add ten' I get 'add nine'

**2<sup>nd</sup> key point:** Commutativity ( $5 + 9 = 9 + 5$ )

### Teaching progression

**Fluency in adding ten:** will already have been secured

Then PPW and practice with number sentences as before

**Adding 8 and 7:** Highlighting possibility of using compensating for adding numbers other than 9 (e.g. 8 and 7)

**Comparison to other strategies:** Highlight that we can also use near doubles and bridging to solve some compensating facts, e.g.  $8 + 9$