St. Edward's Catholic Primary School

Mathematics Calculations Policy

2024/25

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At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. Children should all have access to their age-appropriate curriculum content and, rather than being extended learning accelerating through the national curriculum, they should deepen their conceptual understanding by tackling varied and challenging problems. Similarly, with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these principles and concepts through the use of concrete materials and pictorial representations to ensure fluency and depth of understanding.

The rationale of the concrete-pictorial-abstract (CPA) approach is that for pupils to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. Pupils who grasp concepts rapidly should be challenged through rich and sophisticated problems before any acceleration through new content. Those pupils who are not sufficiently fluent with earlier material should consolidate their understanding, including additional practice, before moving on.

There is also an emphasis placed on instant recall of number bonds and times tables. These need to be mastered to aid with calculations and more challenging problems in readiness for the Multiplication Test at the end of Year 4.

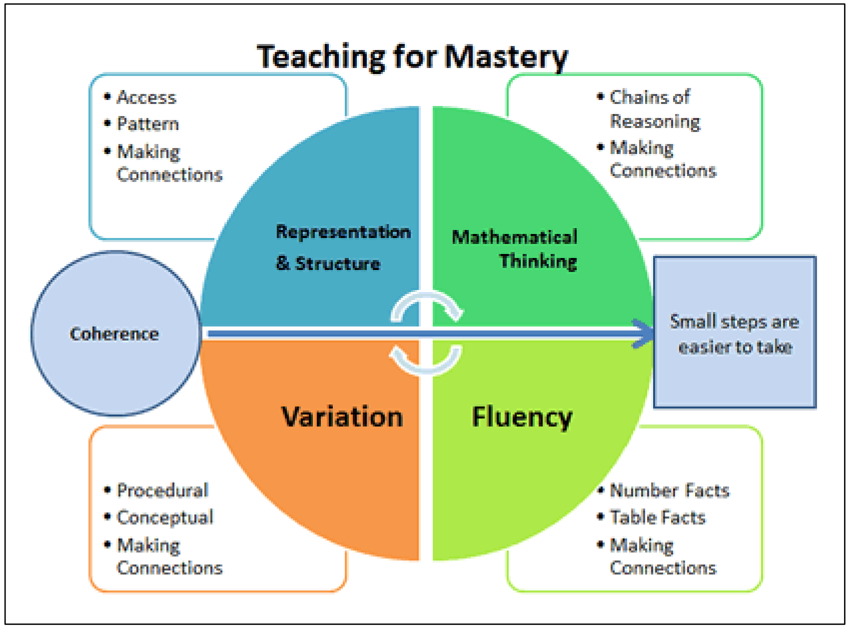
This document outlines the progression of different calculation strategies that should be taught and used from Reception – Year 6, in line with the requirements of the 2014 Primary National Curriculum.

This guidance is to ensure that teachers and parent/carers are aware of the progression of strategies that pupils are formally taught that will support them to perform mental and written calculations. In addition, it will support teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding. We have assigned objectives to year groups based upon guidance from the NCETM. However, it is important to remember that it may sometimes be necessary to revisit strategies from previous year groups if children are working below age related expectations.

This guidance only details the strategies; teachers must plan opportunities for pupils to apply these. Concrete materials shown here are for exemplification; there are many other resources which can be used to aid pupil understanding.

Ideas and images have been derived from NCETM Curriculum Prioritisation Materials, which encompass the Spine Materials and the Ready to Progress Materials, NCTEM Mastering Number and The White Rose Calculation Policy and resources.

This calculation progression document is based on key mastery principles as outlined in the diagram below:



Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts.

It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully.

High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

> The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.
>
> 2014 Maths Programme of Study

Correct	Incorrect
ones	units
Is equal to	equals
Zero	(oh) The letter o

The expectation is that pupils will, at all times, answer in full sentences.

Sentence stems can be used to help pupils with this if required.

Acceptable	Unacceptable
Seven add three is equal to ten.	10
Three groups of five is fifteen.	15
Two and three makes five, Five is made of two and three.	5

say, you say, you say, you say, we all say

This technique enables the teacher to provide a sentence stem for children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. For example: If the rectangle is the whole, the shaded part is one third of the whole. Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge.

Addition and Subtraction

Part-Whole (Cherry) Model

Benefits

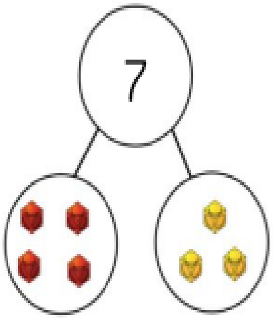
The part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

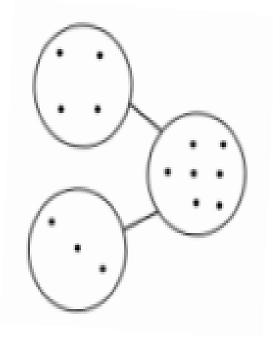
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

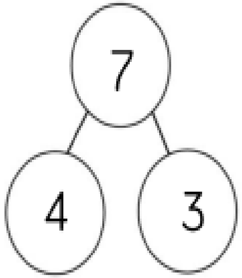
When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract decimals and percentages.





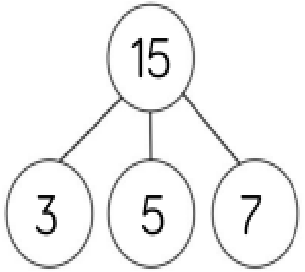


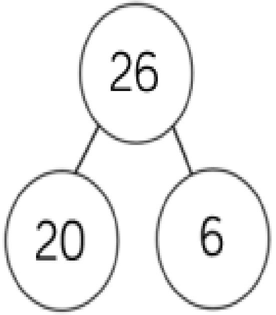
$7 = 4 + 3$

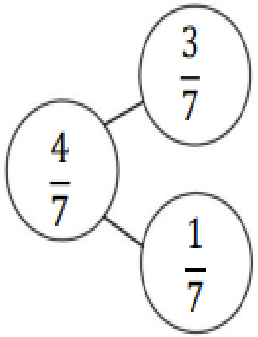
$7 = 3 + 4$

$7 - 3 = 4$

$7 - 4 = 3$







Bar Model (Part- Whole Relationship)

Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

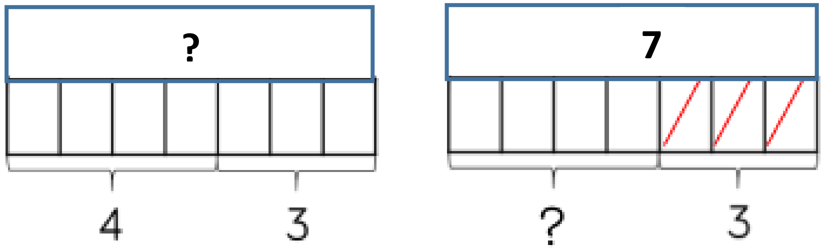
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

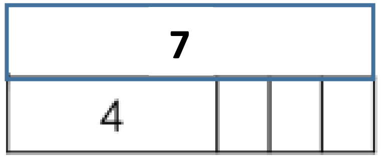
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Concrete

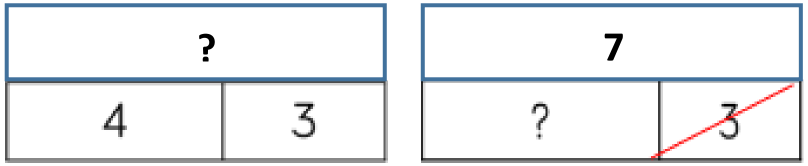
Discrete

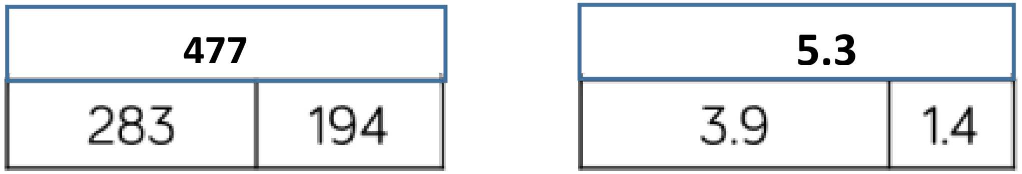


Combination



Continuous





Bar Model (Multiple)

Benefits

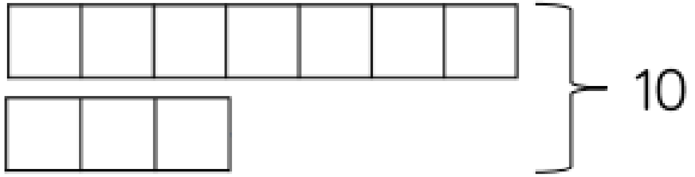
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right-hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

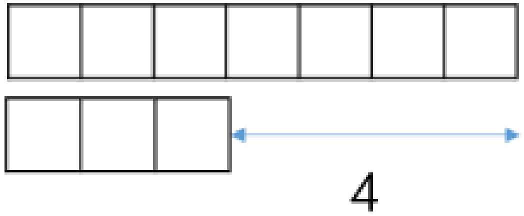
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Discrete

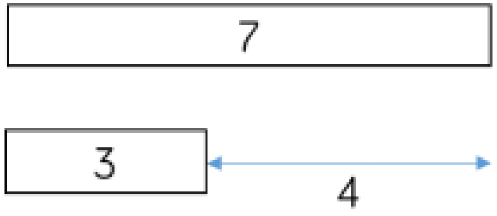


$7 + 3 = 10$

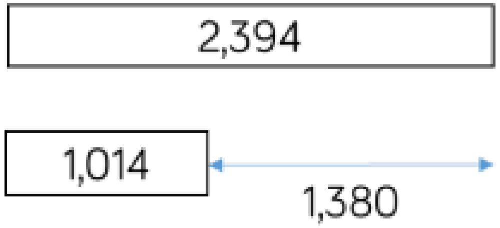


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2,394 - 1,014 = 1,380$

Number Shapes

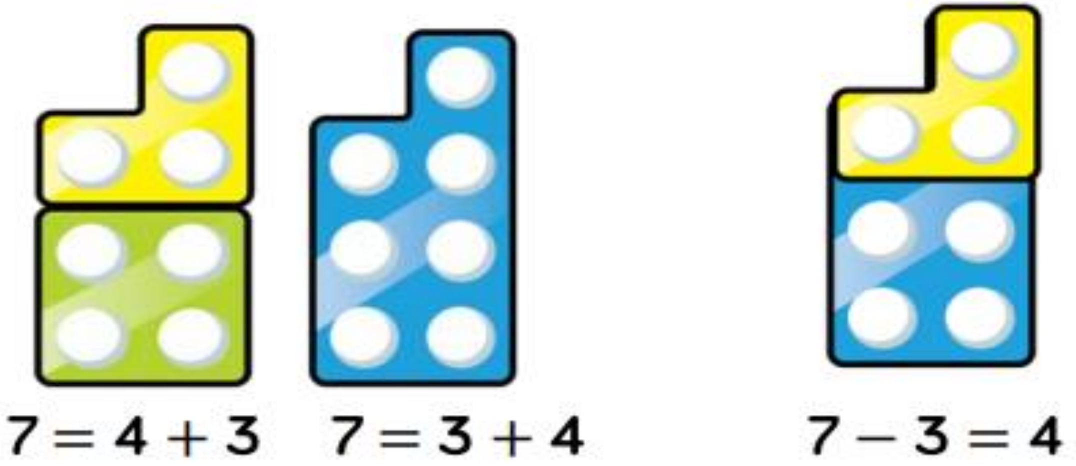
Benefits

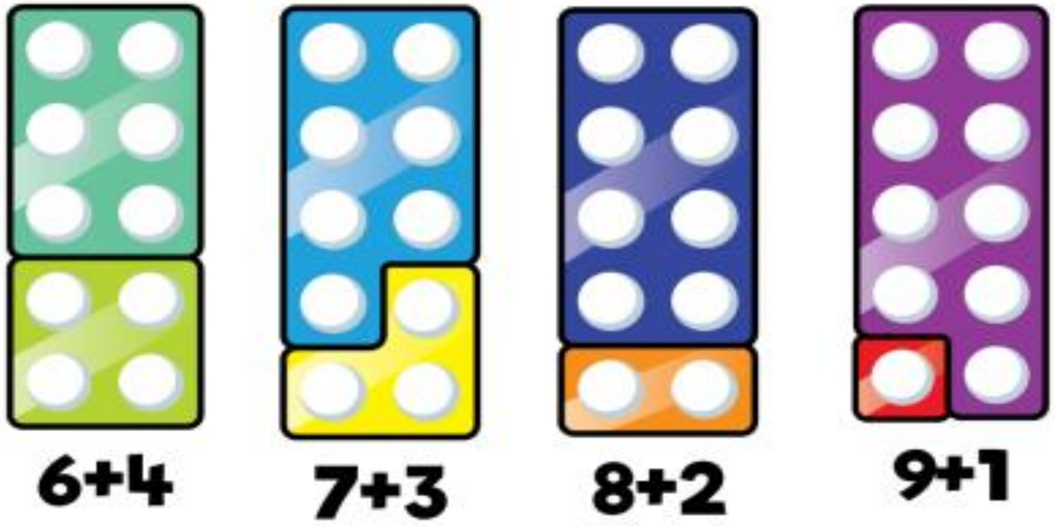
Number shapes can be useful to support children to subitise as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.





Cubes

Benefits

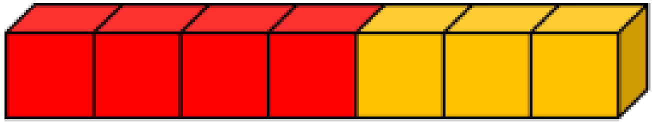
Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

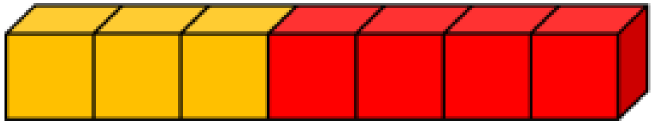
When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

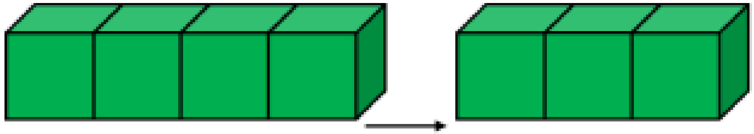
Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.



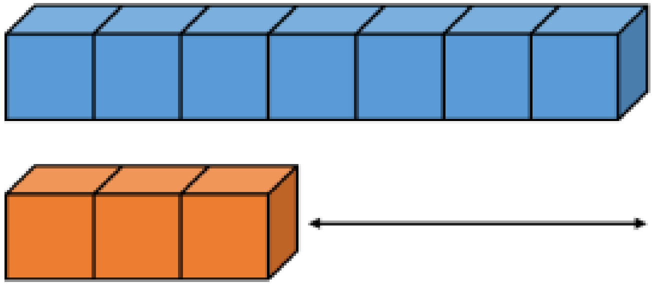
$7 = 4 + 3$



$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

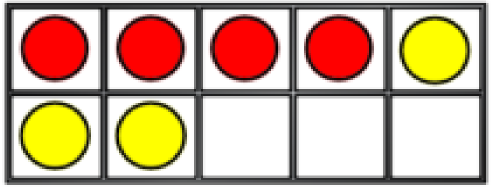
Ten Frames (within 10)

Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

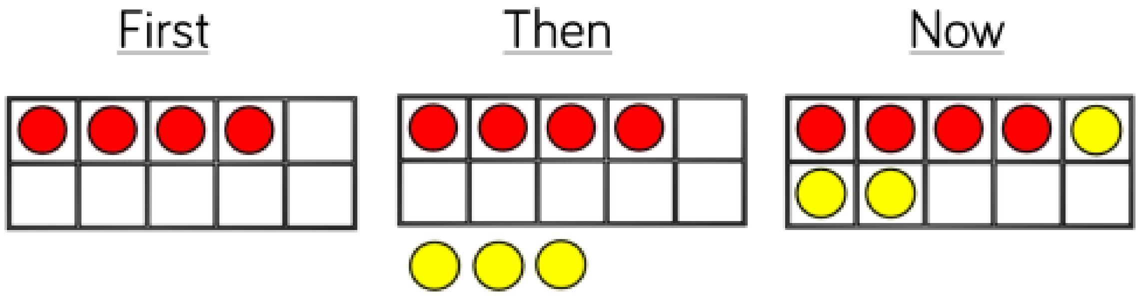
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change eg First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

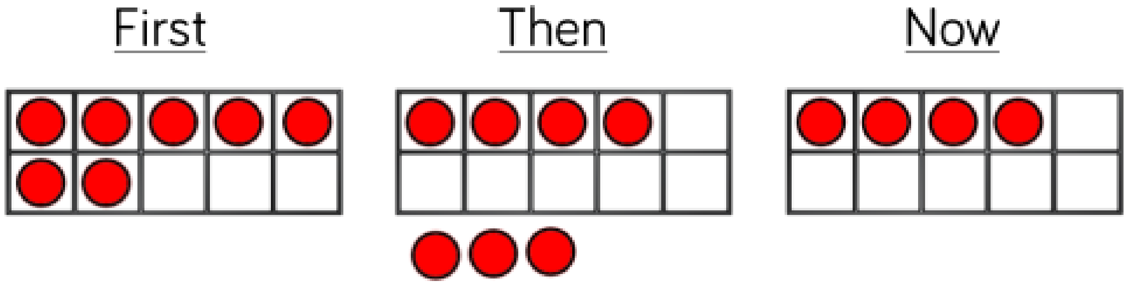


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.



$4 + 3 = 7$



$7 - 3 = 4$

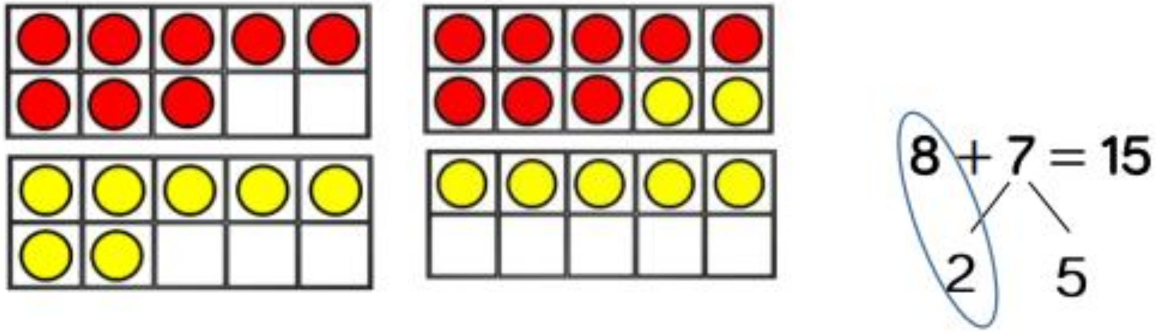
Ten Frames (within 20)

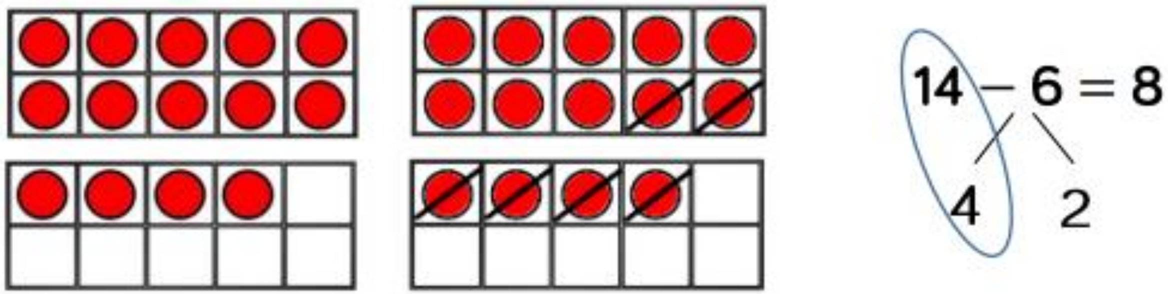
Benefits

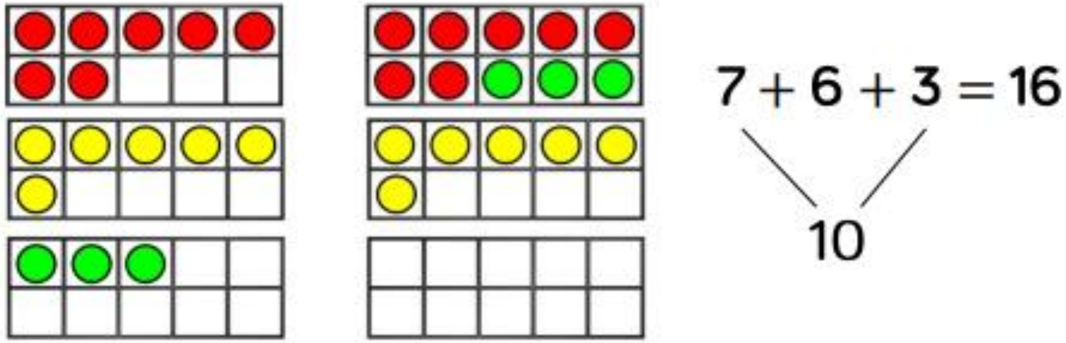
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and make links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding 3 single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. they may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.







Bead Strings

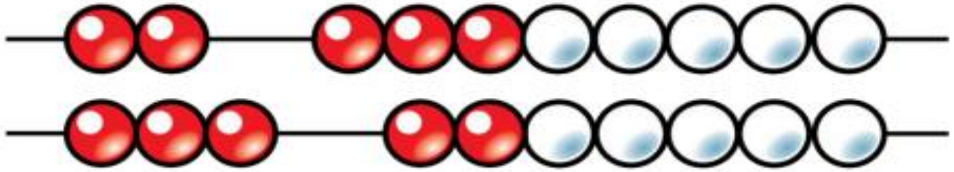
Benefits

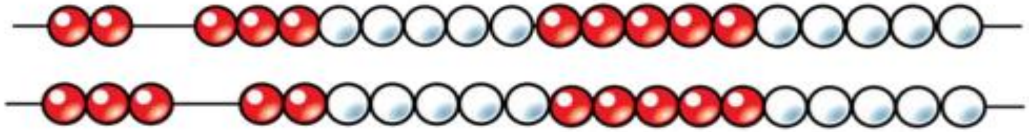
Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into eg 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making 10. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.





Number Tracks

Benefits

Number trcks are useful to support children in their understanding of augmentation and reduction.

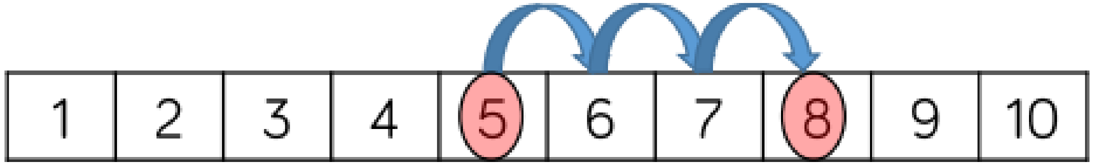
When adding, children count on to find the total of the numbers. On a number track children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

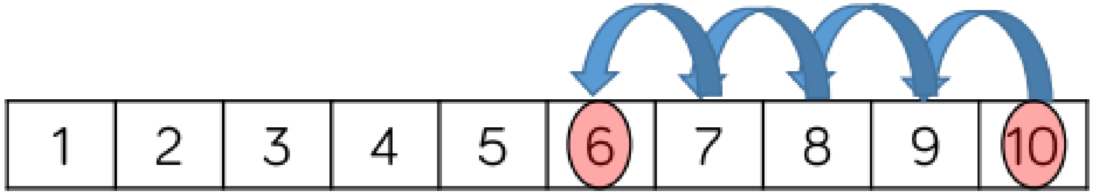
Number tracks can work well alongside ten frames and bead strings which can model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

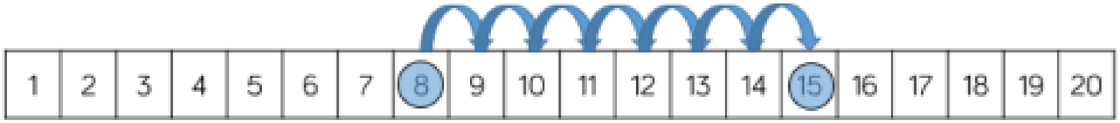
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$



Number Lines (labelled)

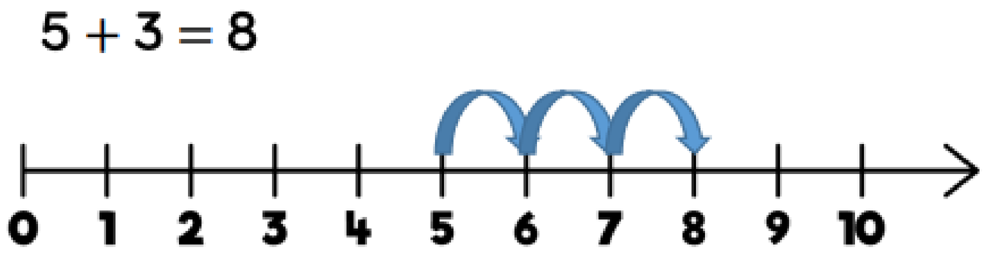
Benefits

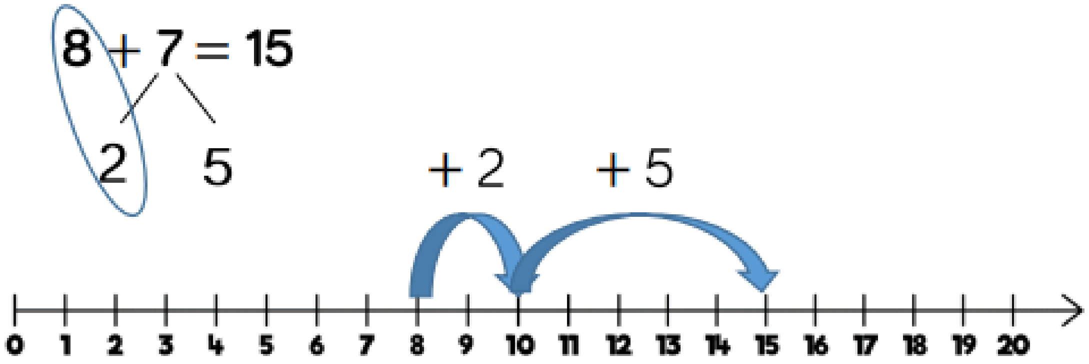
Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

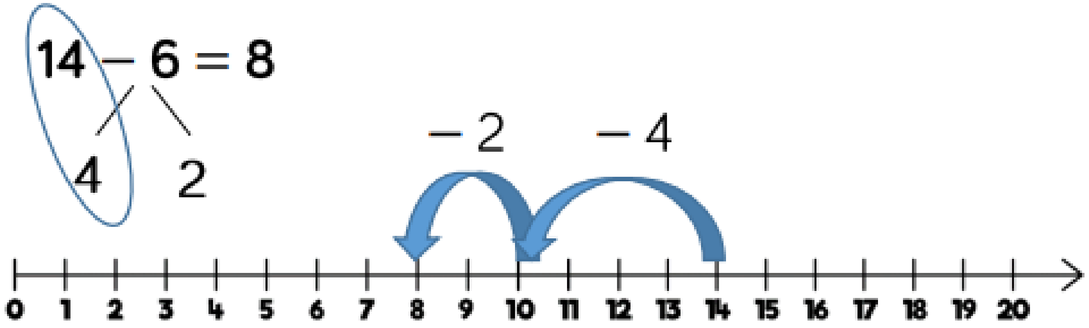
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.







Number Lines (blank)

Benefits

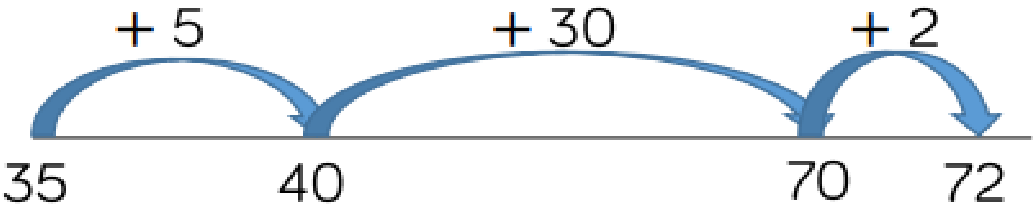
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

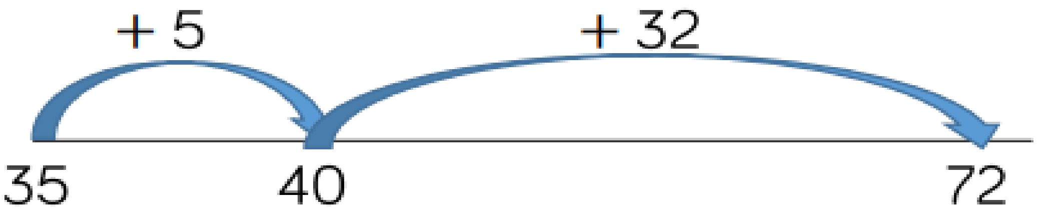
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

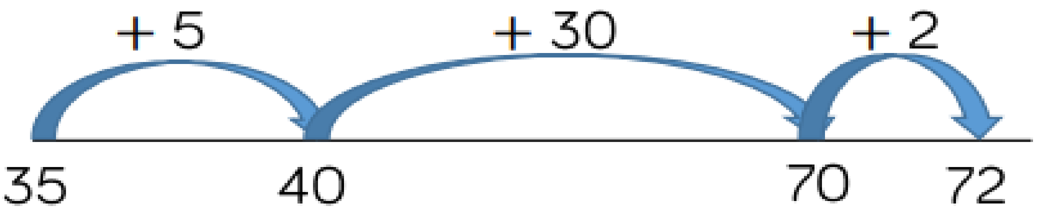
$35 + 37 = 72$



$35 + 37 = 72$



$72 - 35 = 37$



Straws

Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

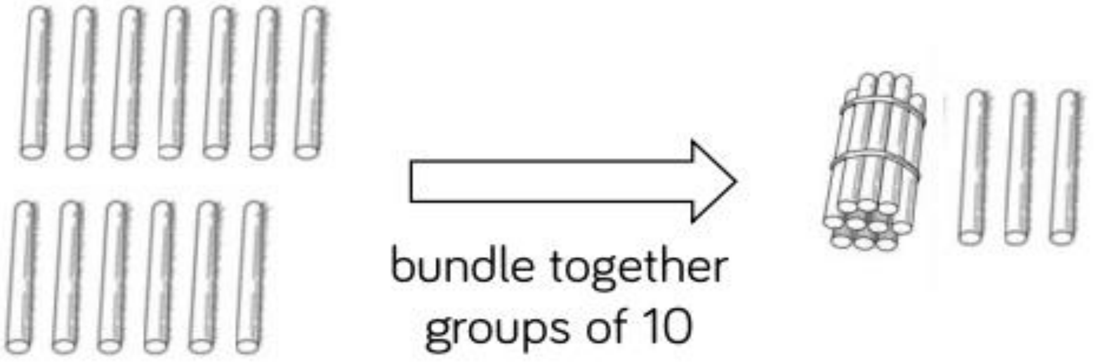
Children can be introduced to the idea of bundling groups of 10 when adding smaller numbers and when representing 2-digit numbers.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

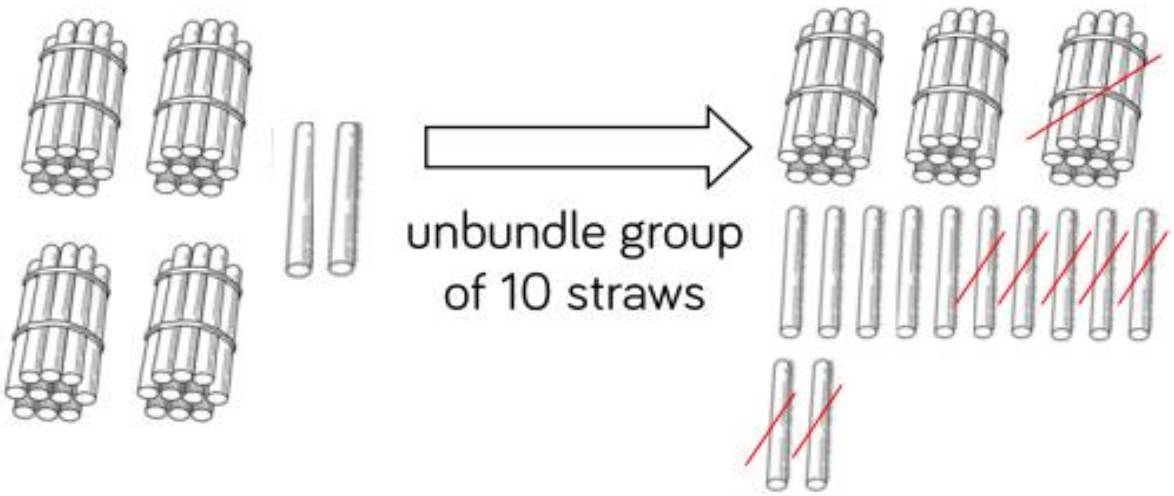
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

$7 + 6 = 13$



$42 - 17 = 25$



Base 10/Dienes (addition)

Benefits

Using Base 10 is an additional way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

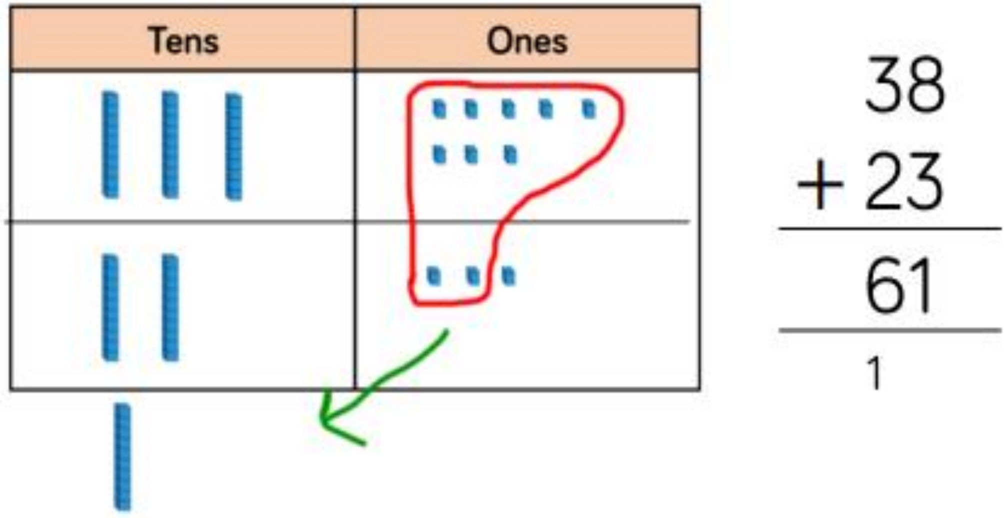
When adding, always start with the smallest place value column. Here are some questions to support children.

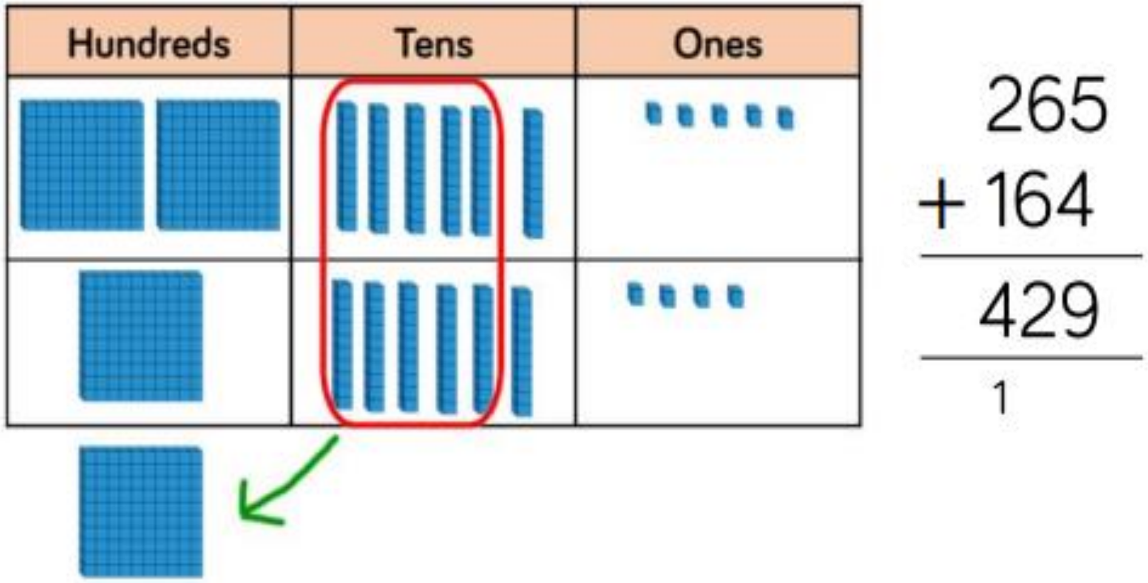
How many ones are there altogether?

Can we make an exchange?

How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)

How many ones do we have left? (write in ones column)





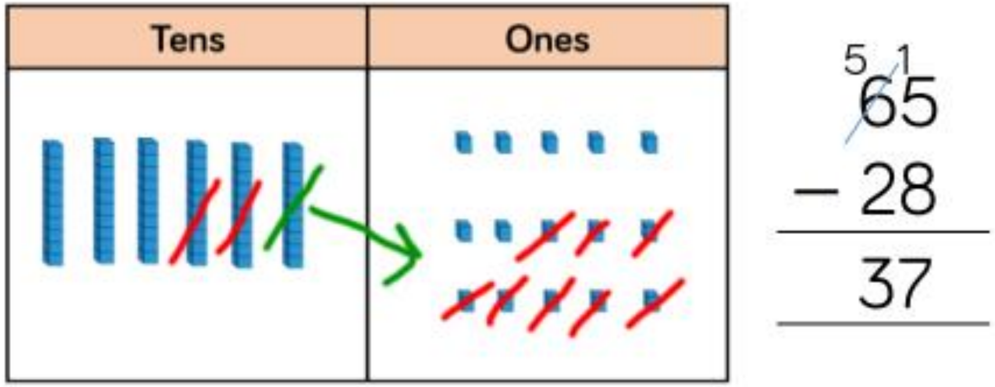
Base 10/Dienes (subtraction)

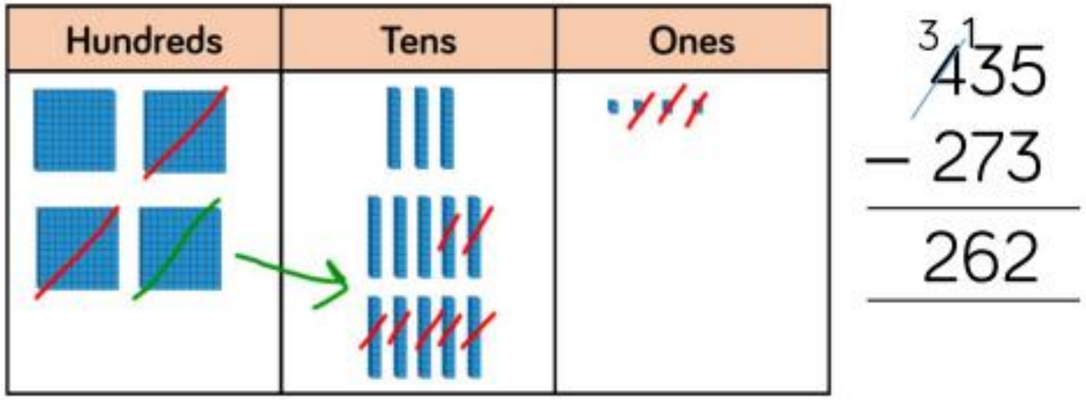
Benefits

Using Base 10 is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left to exchange eg exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.





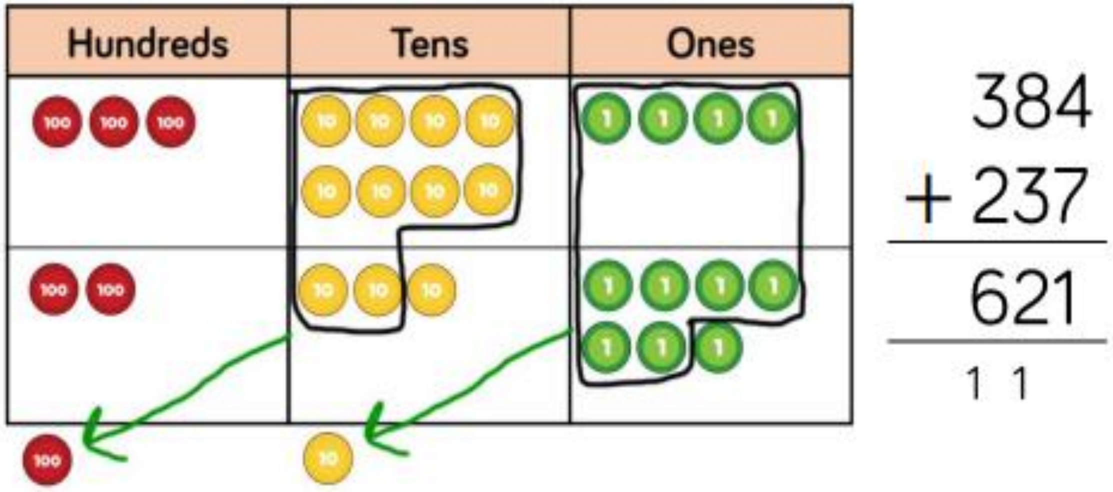
Place Value Counters (addition)

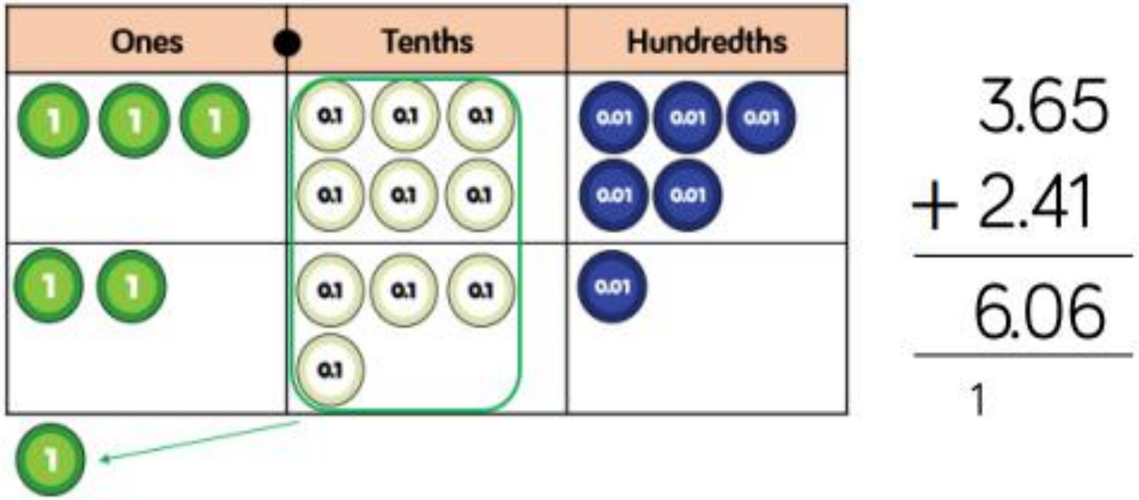
Benefits

Using place value counters is an effective way to support children’s understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so that they can see the clear links between the written method and the model.

Children should first add without exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. They should be used on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.





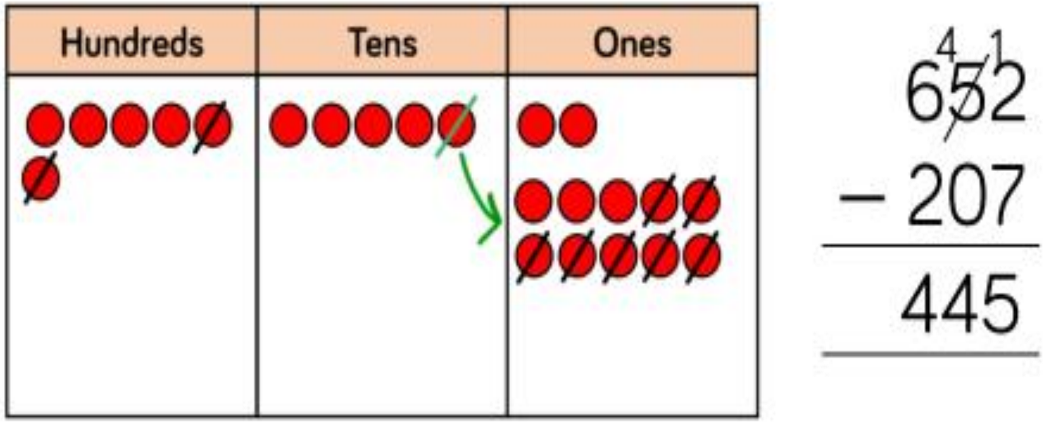
Place Value Counters (addition)

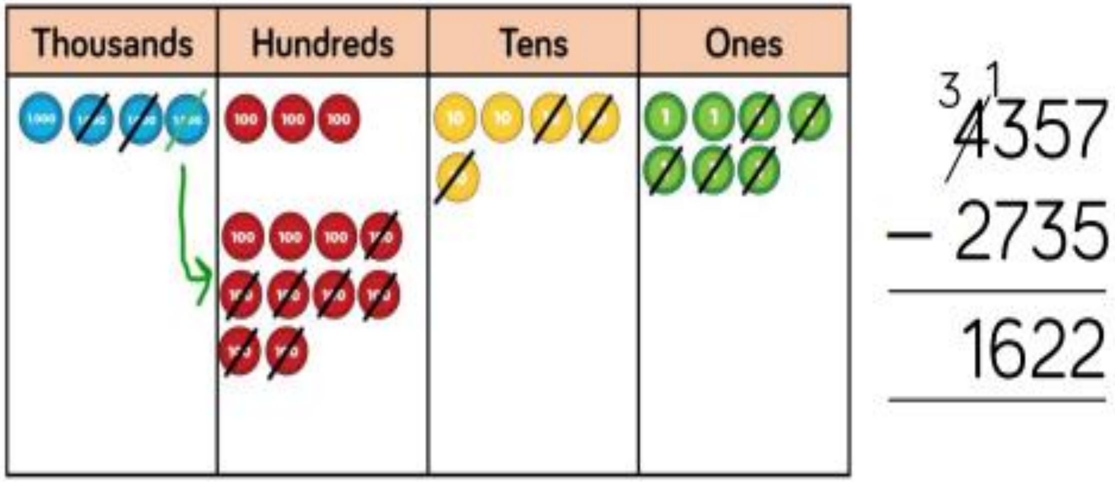
Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so that they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. The counters should be put on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange eg exchange 1 ten for 10 ones. They can then subtract efficiently.





Addition and Subtraction

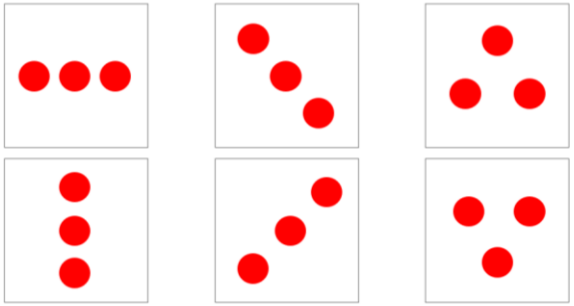
EYFS

Subitise small groups of objects and pictures

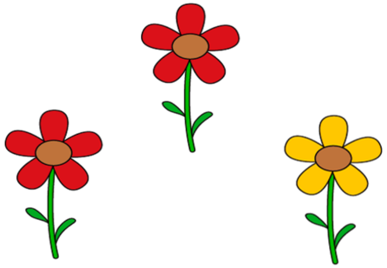
Know one more and one less

Vocabulary:

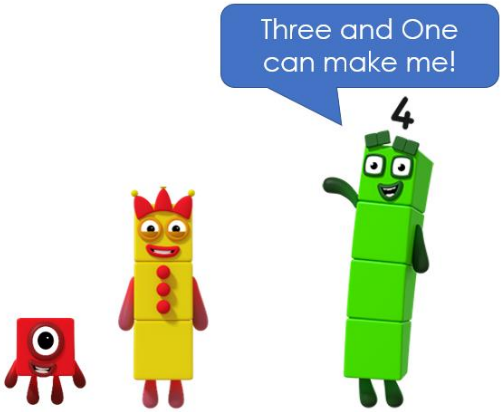
Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Composition Combine Partition Numberblocks Part-Whole (Cherry) Model Tens Frame Fingers Five and-a-bit Systematic Subitise One more One less

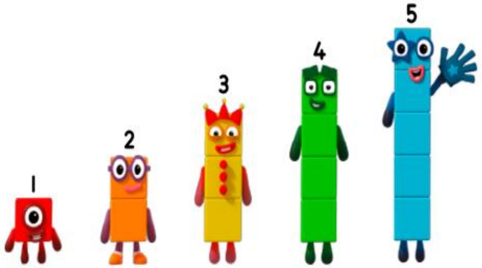


Recognise standard and non-standard arrangements up to 5.



Know that numbers are made up of smaller numbers. E.g. There are 3 flowers, 3 is made of 2 and 1.





Recognise the stair case pattern and the relationship between numbers.

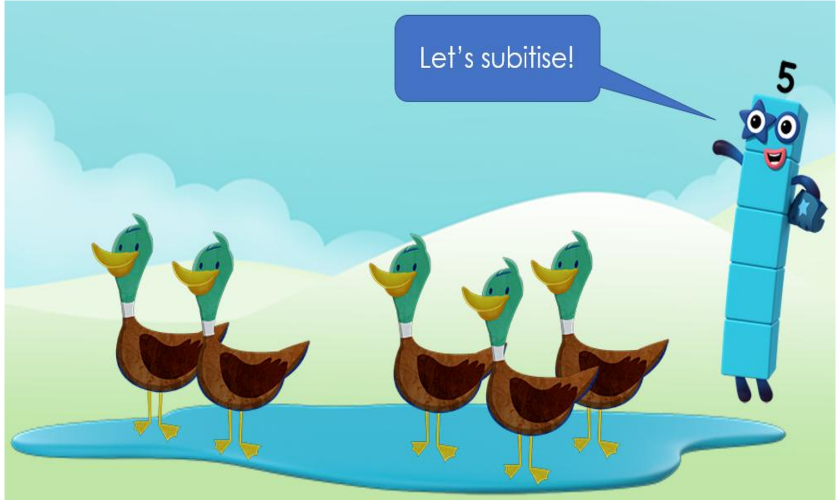
Addition and Subtraction

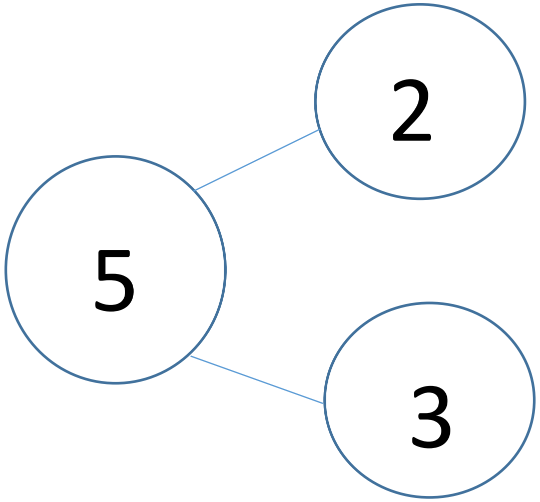
EYFS

Combine and partition groups of objects.

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Numberblocks Part-Part-Whole (Cherry) model Tens Frame Fingers Five and-a-bit Systematic Subitise One more One less





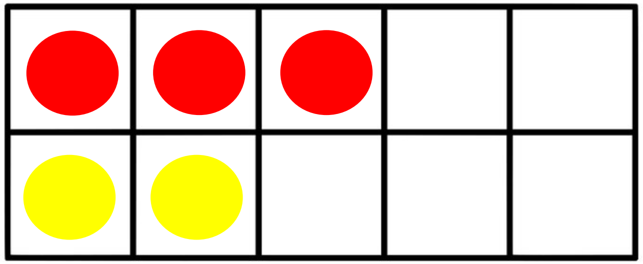
Pictorial representations can also be used in the part whole model.

Combine two parts to make a whole. Represent using tens frames, part whole models and sentences.

2 and 3 make 5.

5 is made from 2 and 3.

2 less than 5 is 3.



Addition and Subtraction

Year 1

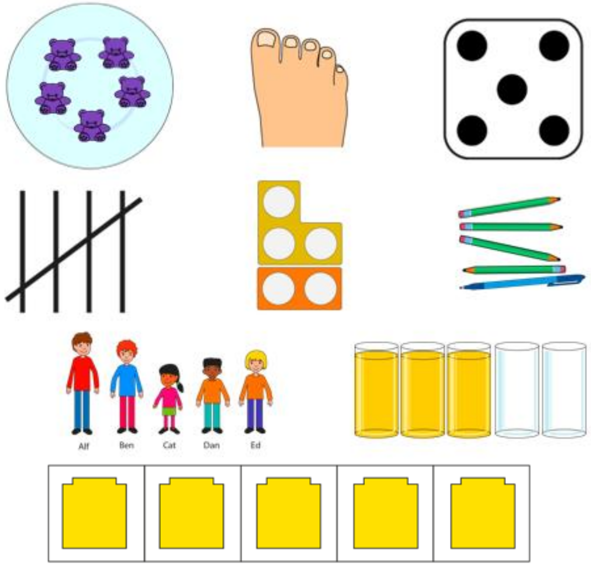
Compose and Partition Numbers to 10 (1)

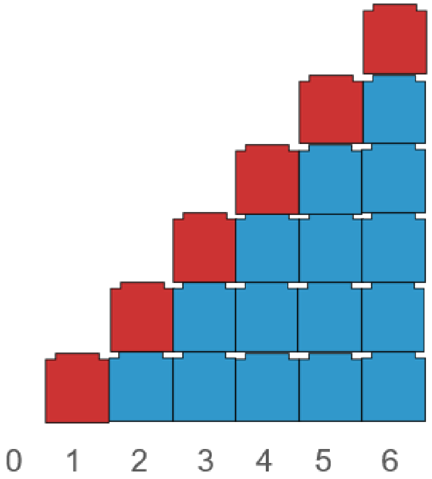
Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Numberblocks Part-Part-Whole (Cherry) model Tens Frame Fingers Five and-a-bit Systematic Subitise One more One less

Understand that numbers to 10 can be represented in many different ways.

Numbers to 5 can be identified without counting (subitising).



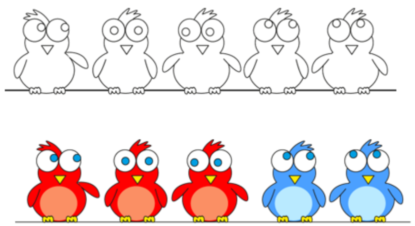


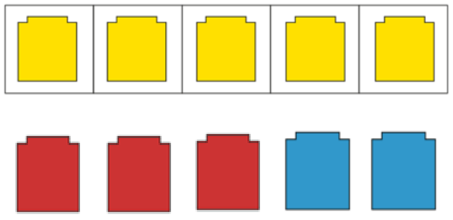
Each number is composed of the previous number and one more.

A number can be partitioned in different ways.

There are 5 ____. 3 are ____. 2 are ____.

5 is the whole. 3 is a part. 2 is a part.

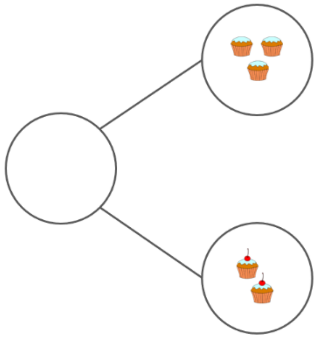


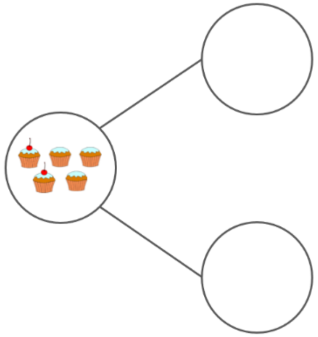


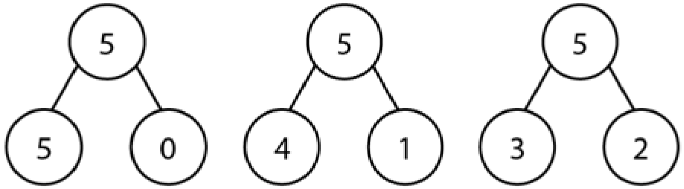
Each number can be partitioned into two smaller numbers

There are 5 ____. 3 are ____. 2 are ____.

5 is the whole. 3 is a part. 2 is a part.







Addition and Subtraction

Year 1

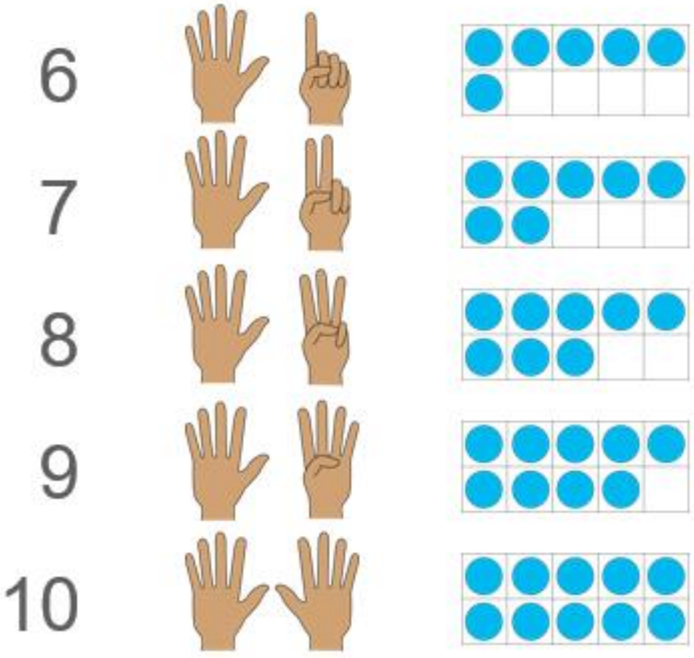
Compose and Partition Numbers to 10 (2)

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten Represents Compose Combine Partition Numberblocks Part-Part-Whole (Cherry) model Tens Frame Fingers Five and-a-bit Systematic Subitise One more One less

	Blue	**Red**
🔴🔴🔴🔴🔴	0	5
🔵🔴🔴🔴🔴	1	4
🔵🔵🔴🔴🔴	2	3
🔵🔵🔵🔴🔴	3	2
🔵🔵🔵🔵🔴	4	1
🔵🔵🔵🔵🔵	5	0

A number can be partitioned in different ways systematically.



Numbers from 6 – 10 are composed of the '5 and a bit' structure.

Addition and Subtraction

Year 1

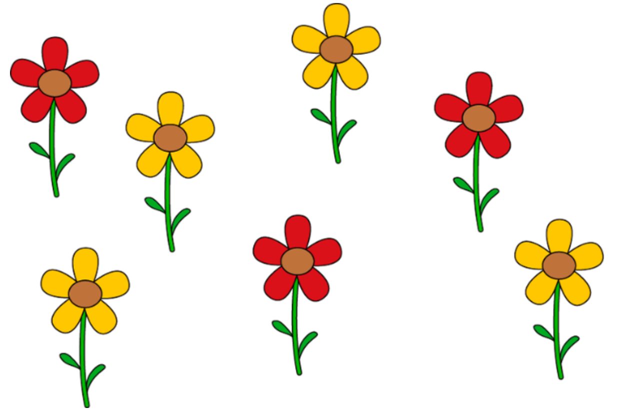
Read, Write and Interpret Additive Equations (1)

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation

Addend + Addend = Sum

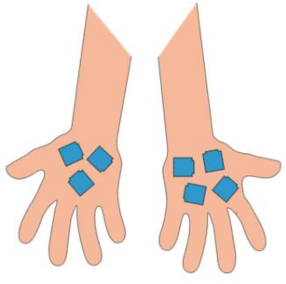
Minuend – Subtrahend = Difference



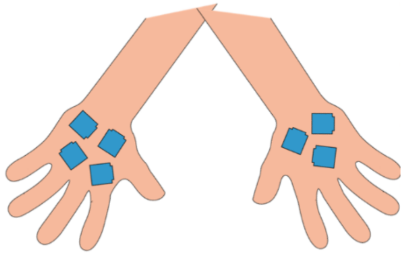
Identify what each number represents in an expression.

The 4 represents the 4 yellow flowers.

The 3 represents the 3 red flowers.



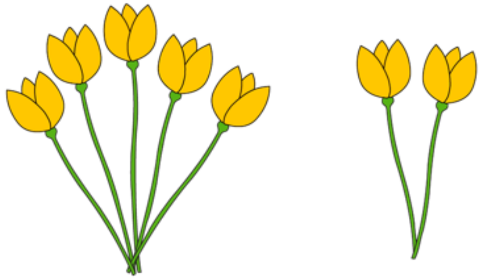
3 + 4



4 + 3

We can write the addends in either order.

(Commutative Law)



5 + 2 = 7

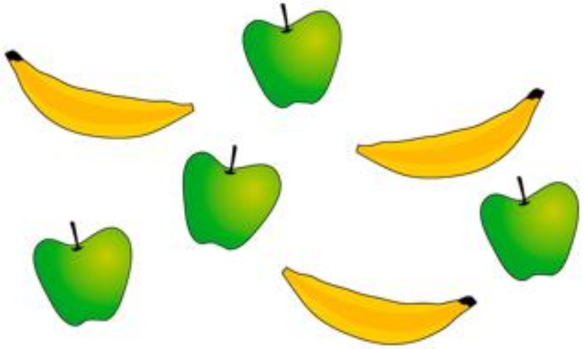
Identify what each number represents in an expression.

We can write 5 plus 2 is equal to 7.

The 5 represents ____.

The 2 represents___.

The 7 represents the total number of ___.



4 + 3 = 7

Addition and Subtraction

Year 1

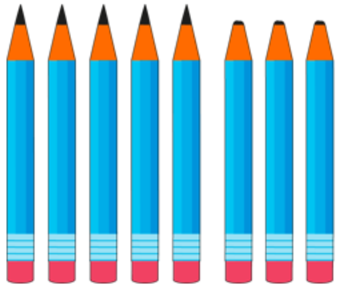
Read, Write and Interpret Additive Equations (2)

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation

Addend + Addend = Sum

Minuend – Subtrahend = Difference



Subtraction can tell us about partitioning.

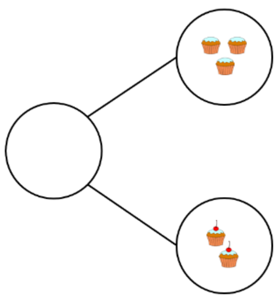
There are 8 ____ altogether.

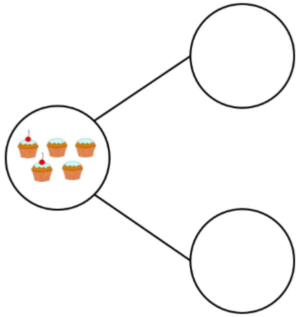
5 ___ are ____.

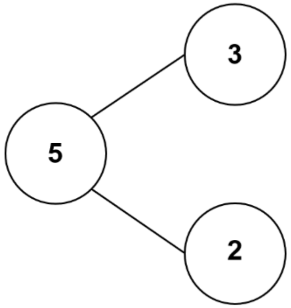
3 ___ are ____.

We can write this as 8 minus 5 is equal to 3.

8 – 5 = 3







Make connections between addition and subtraction using the part-part-whole model.

Addition can tell us about combining objects.

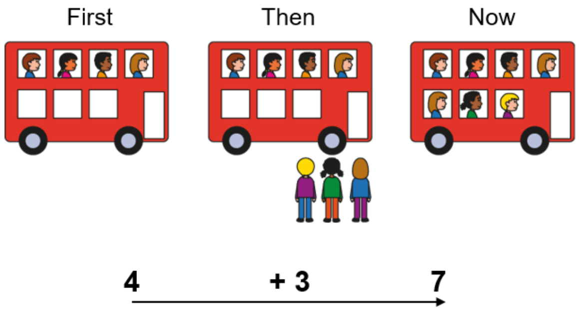
Subtraction can tell us about partitioning objects.

2 + 3 = 5

3 + 2 = 5

5 – 3 = 2

5 – 2 = 3

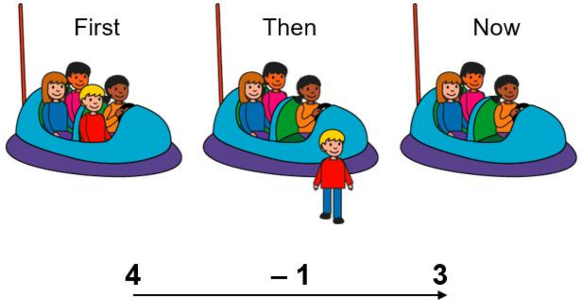


4 + 3 = 7

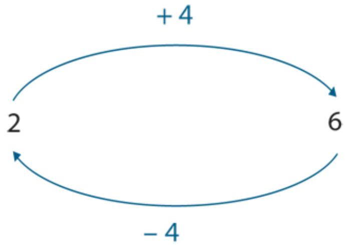
Understand the First, Then, Now structure of addition and subtraction.

Addition can tell us about a quantity increasing.

Subtraction can tell us about a quantity decreasing.



4 – 1 = 3



Addition and Subtraction undo each other.

Addition and Subtraction

Year 2

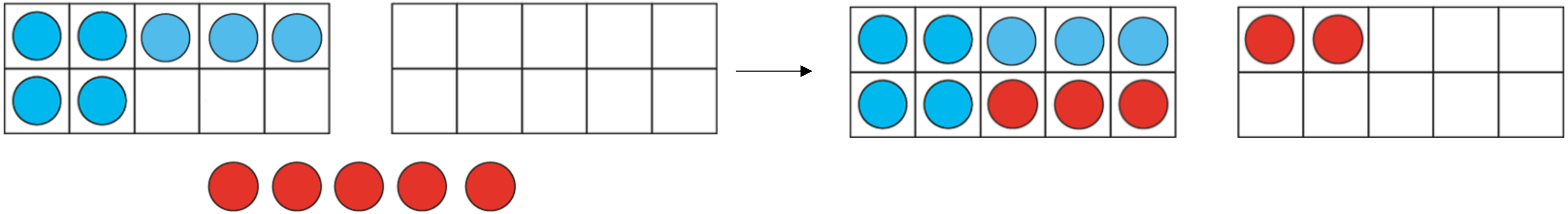
Add and Subtract across 10 (1)

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation

Addend + Addend = Sum

Minuend – Subtrahend = Difference



7 + 5

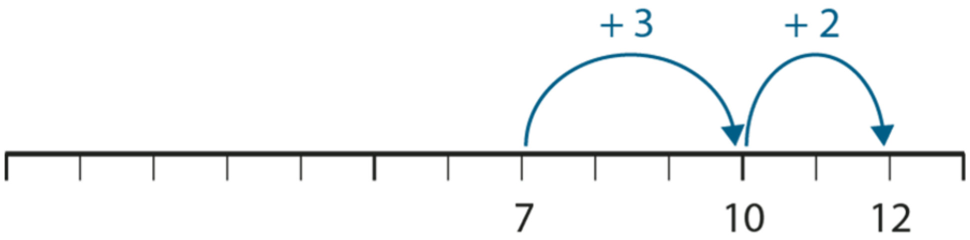
7 + 5 = 7 + 3 + 2 = 10 + 2

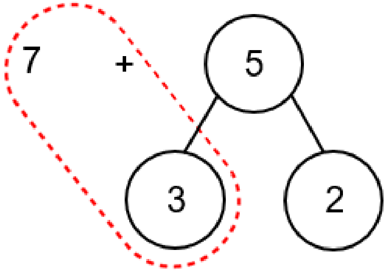
Use knowledge of known facts to bridge 10 using a 'make 10' strategy.

First, I partition the __ into ___ and ___.

Then, I add ___ and ___ to make 10.

Then, I add the remaining ___ to make ___.





7 + 3 = 10

10 + 2 = 12

Addition and Subtraction

Year 2

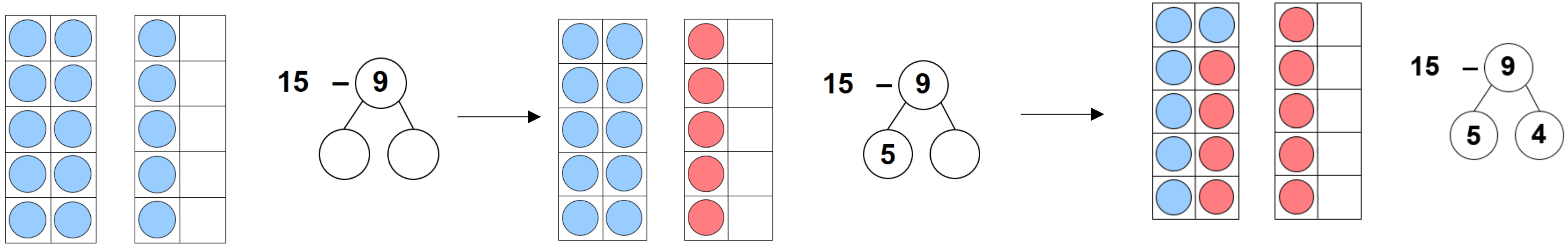
Add and Subtract across 10 (2)

Vocabulary:

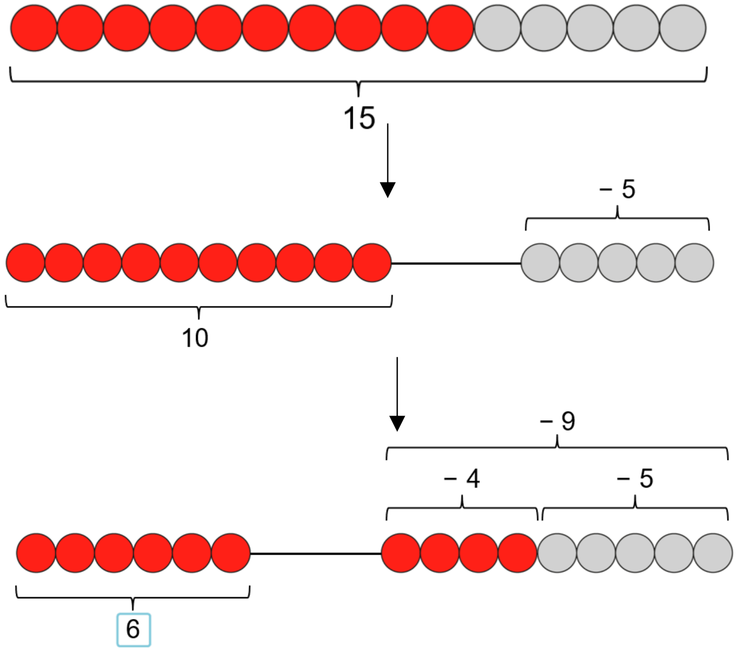
Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation

Addend + Addend = Sum

Minuend – Subtrahend = Difference



15 – 9 = 6

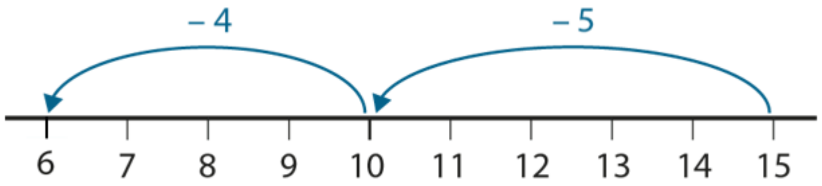


Use knowledge of known facts to subtract *through 10*. We can partition the subtrahend to help us subtract.

First, I partition the __ into ___ and ___.

Then, I subtract ___ and ___ to get to 10.

Then, I subtract the remaining ___ to make ___.



Addition and Subtraction

Year 2

Add and Subtract across 10 (3)

Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation

Addend + Addend = Sum

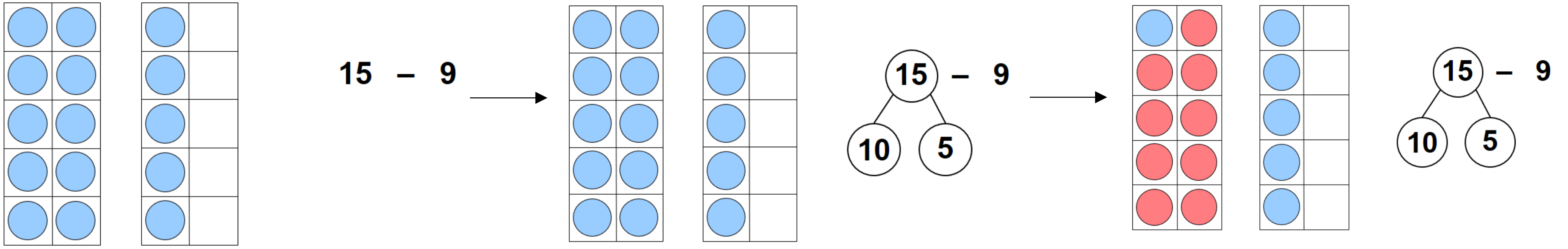
Minuend – Subtrahend = Difference

Use knowledge of known facts to subtract *from 10*. We can partition the subtrahend to help us subtract.

First, I partition the __ into ___ and ___.

Then, I subtract ___ from 10 to make ___.

Then, I add the remaining ___ to make ___.



10 – 9 = 1

1 + 5 = 6

15 – 9 = 6

Addition and Subtraction

Year 2

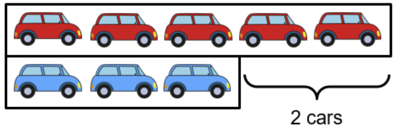
Solve Comparative Addition and Difference Problems

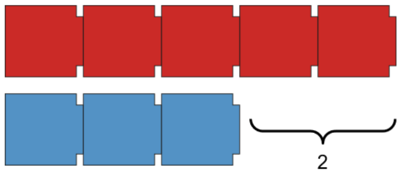
Vocabulary:

Part Whole One Two Three Four Five Six Seven Eight Nine Ten
Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model
Tens Frame Fingers Five and-a-bit Systematic Plus + Minus - Equal to =
Addition Subtraction Quantity Increase Decrease First, Then, Now
Expression Equation Difference Bar model

Addend + Addend = Sum

Minuend – Subtrahend = Difference

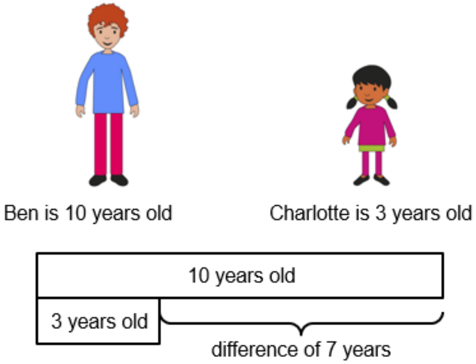




Line up sets of objects in a bar model structure to support comparison.

There are 2 fewer blue cars than red cars.

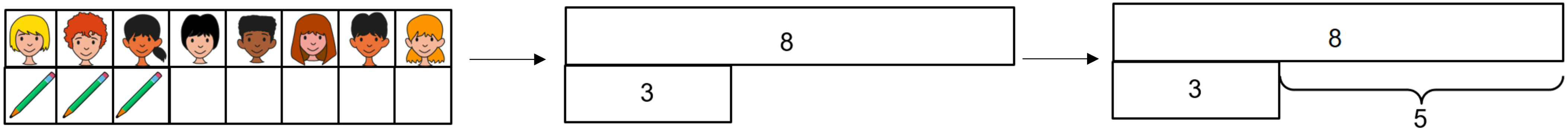
There are 2 more red cars that blue cars.



Represent a range of comparison contexts.

Ben is 7 years older than Charlotte.

Charlotte is 7 years younger than Ben.

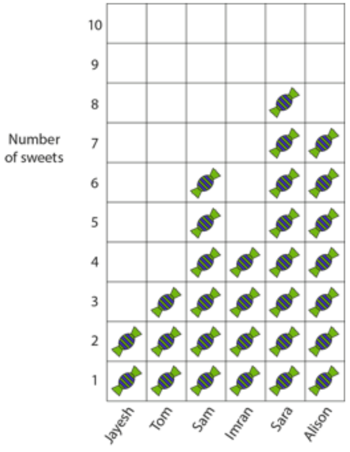


We can use subtraction to help solve difference problems / missing addend problems about 'how many more?' and 'how many fewer?'

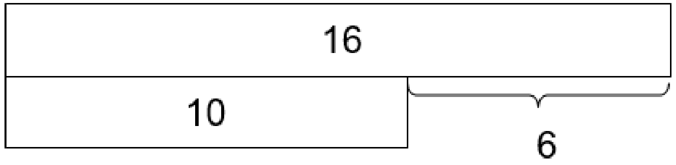
3 + __ = 8

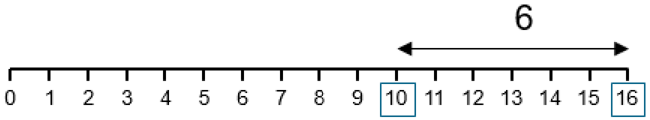
8 – 3 = 5

Create contexts for recognising the difference/comparative addition structure with all representations below.



$10 + \boxed{} = 16$ **$16 - 10 = \boxed{}$**





Addition and Subtraction

Year 2

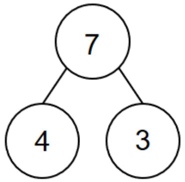
Add and Subtract within 100 (1).

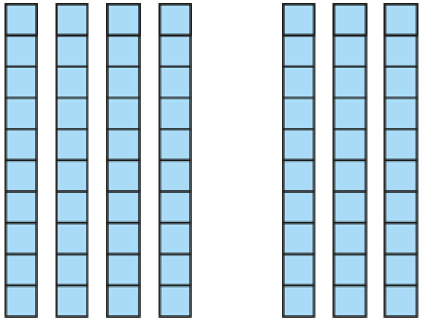
Vocabulary:

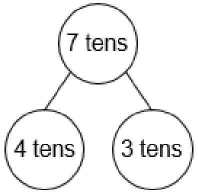
Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole (Cherry) model Tens Frame Deines Plus + Minus - Equal to =
Addition Subtraction Expression Equation Exchange Count on Count back
Number line Tens Boundary

Addend + Addend = Sum

Minuend – Subtrahend = Difference







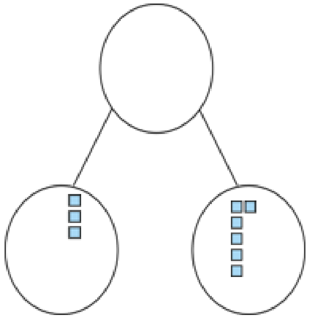
Use known facts within 10 to add/subtract multiples of 10.

I know that 4 plus 3 is equal to 7.

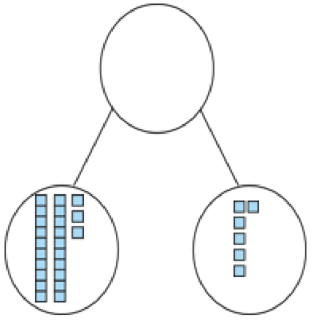
So, 4 tens plus 3 tens is equal to 7 tens.

40 + 30 = 70.

70 – 40 = 30



3 + 6 = 9



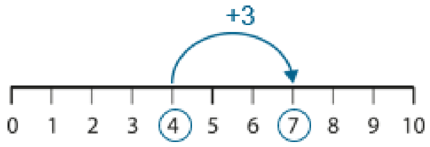
23 + 6 = 29

Use known facts within 10 to add/subtract ones to/from a 2 digit number.

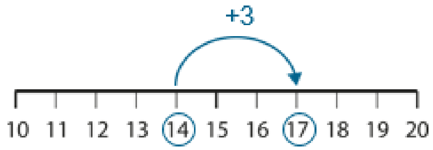
I know that 3 plus 6 is equal to 9.

So, 2 tens and 3 ones plus 6 ones is equal to 2 tens and 9 ones.

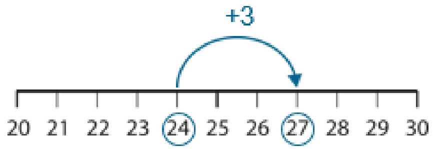
23 + 6 = 29.



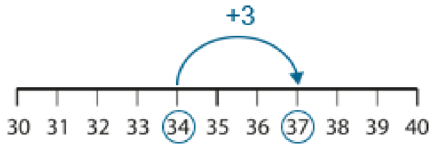
4 + 3 = 7



14 + 3 = 17



24 + 3 = 27



34 + 3 = 37

Generalise that adding/subtracting within 10 can be applied to adding a 2 digit number with a 1 digit number – not crossing the tens boundary.

I know that 4 plus 3 is equal to 7.

So, 1 ten and 4 ones plus 3 ones is equal to 1 tens and 7 ones.

14 + 3 = 17.

Addition and Subtraction

Year 2

Add and Subtract within 100 (2).

Vocabulary:

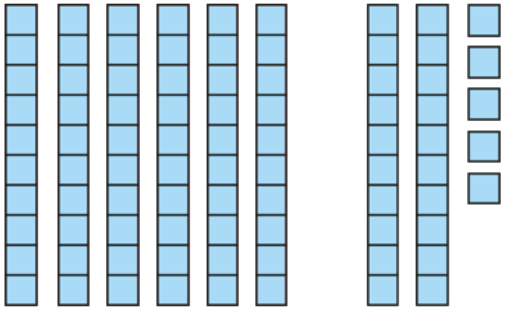
Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole (Cherry) model Tens Frame Deines Plus + Minus - Equal to =
Addition Subtraction Expression Equation Exchange Count on Count back
Number line Tens Boundary

Addend + Addend = Sum

Minuend – Subtrahend = Difference

6 + 2 = 8

60 + 25 = ?



Use known facts within 10 to add/subtract multiples of 10 to a 2 digit number.

I know that 6 plus 2 is equal to 8.

So, 6 tens plus 2 tens is equal to 8 tens. Then add the additional 5 ones.

60 + 25 = 85.

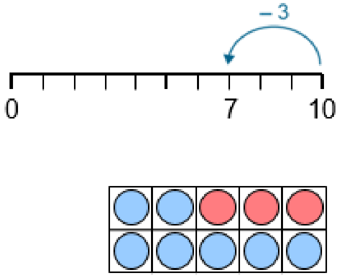
Use knowledge of subtracting from 10 to subtract a single-digit number from a multiple of 10.

I know that 10 minus 3 is equal to 7.

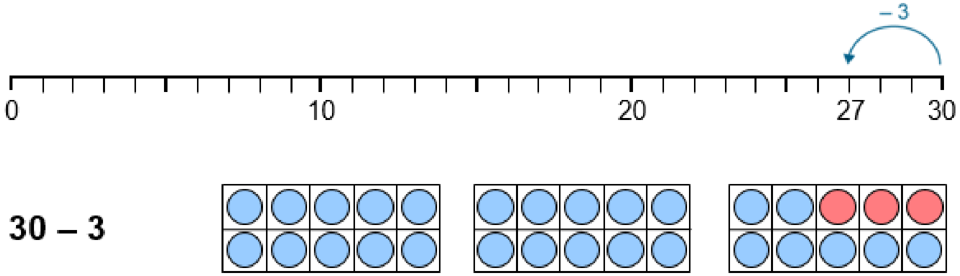
So, 3 tens minus 3 ones is equal to 2 tens and 7 ones.

30 - 3 = 27.

10 – 3



30 – 3



Addition and Subtraction

Year 2

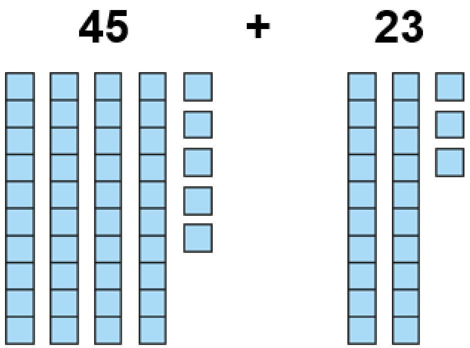
Add and Subtract within 100 (3).

Vocabulary:

Part Whole Ones Tens Represents Compose Combine Partition Total Part-Part-Whole (Cherry) model Tens Frame Deines Plus + Minus - Equal to = Addition Subtraction Expression Equation Exchange Count on Count back Number line Tens Boundary

Addend + Addend = Sum

Minuend – Subtrahend = Difference

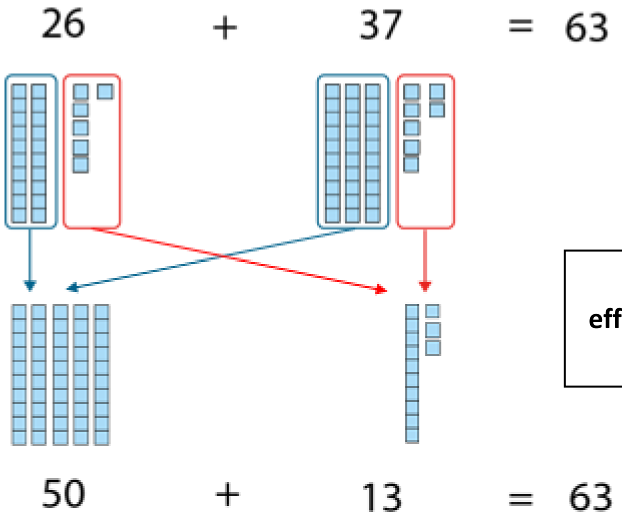


40 + 20 = 60

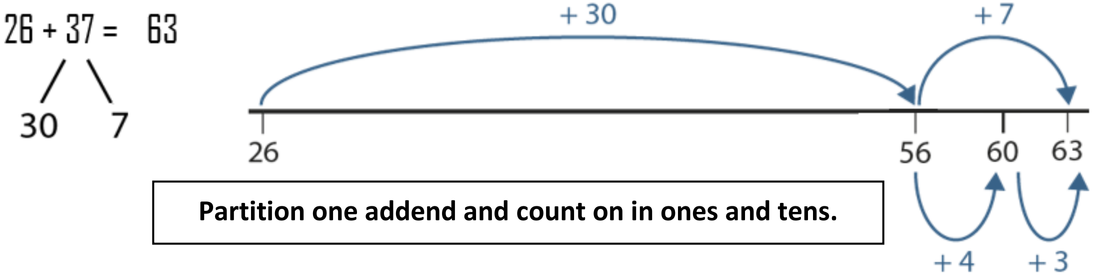
5 + 3 = 8

60 + 8 = 68

Partition both addends to add efficiently without crossing the tens boundary.

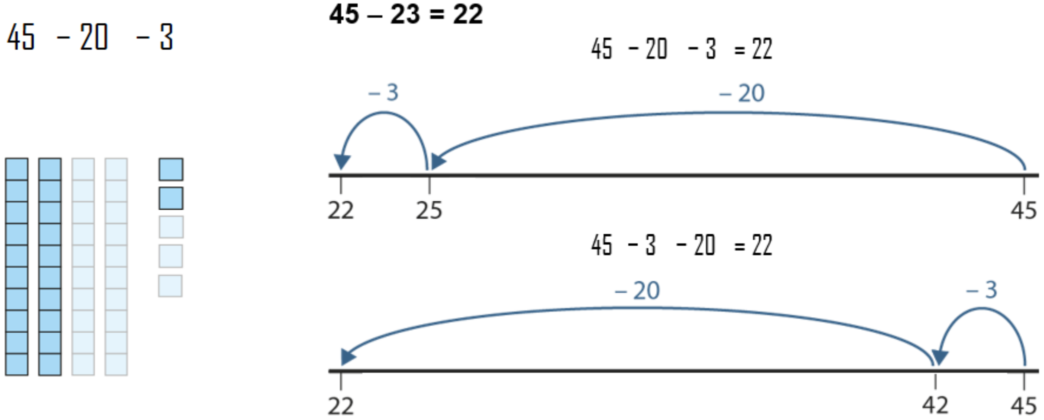


Partition both addends to add efficiently when the ones require an exchange.

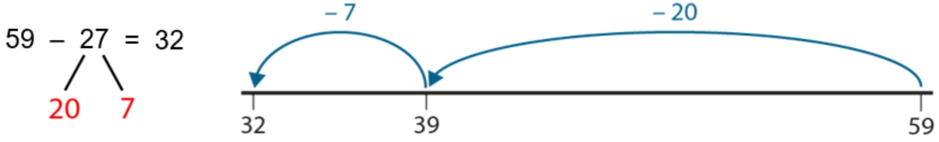


Partition one addend and count on in ones and tens.

45 – 23 = 22



Subtract from any two-digit number by subtracting tens then ones without crossing a tens boundary.



Subtract from any two-digit number by portioning the subtrahend into tens and ones and counting back.

Addition and Subtraction

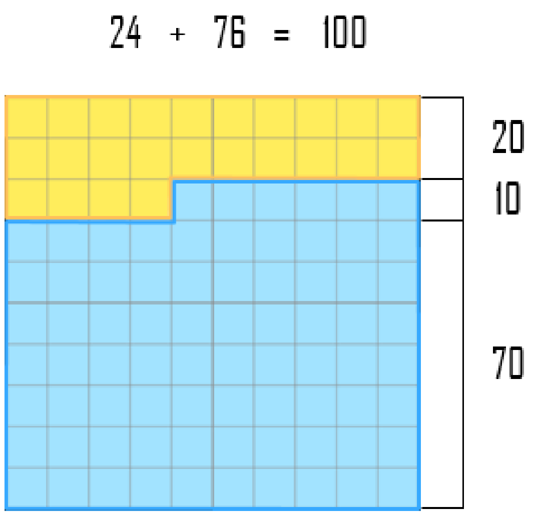
Year 3

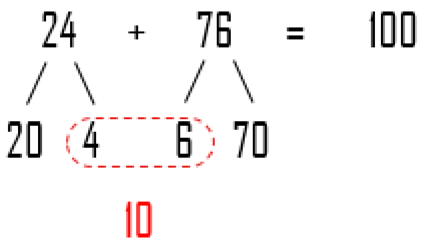
Calculate complements to 100.

Vocabulary:

Part Whole Ones Tens Represents Compose Combine Partition Total
Part-Part-Whole (Cherry) model Deines 100 square Plus + Minus - Equal to =
Addition Subtraction Expression Equation Exchange Complements

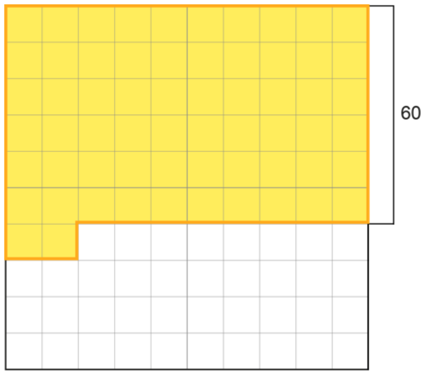
Addend + Addend = Sum





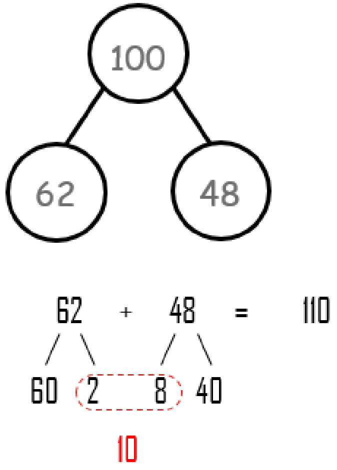
Use knowledge of subtracting from 10 to subtract a single-digit number from a multiple of 10.

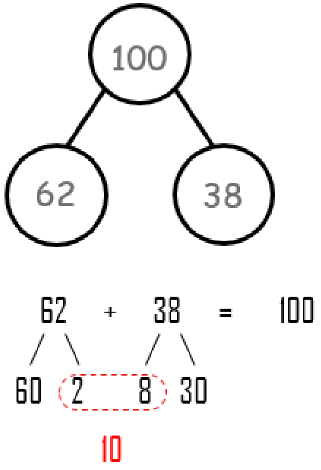
First we make 10 ones. The ones digits add up to make 1 ten, so we need 9 more tens to make a total of 100.



62 + ☐ = 100

Solve missing number problems that sum to 100.





Compare equations which do and do not sum to 100.

Addition and Subtraction

Year 3

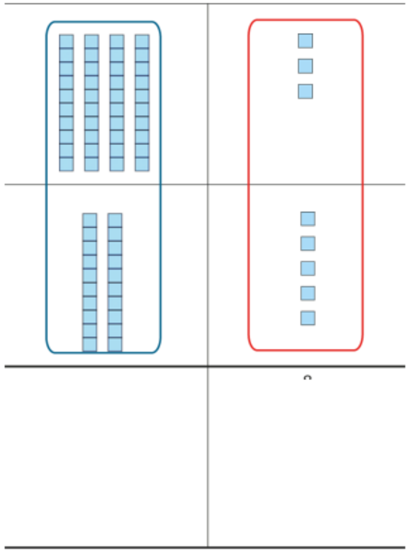
Columnar Addition and Subtraction

Vocabulary:

Ones Tens Represents Compose Combine Total Deines Plus + Minus -
Equal to = Addition Subtraction Equation Regroup Algorithm

Addend + Addend = Sum

Minuend – Subtrahend = Difference



	10s	1s
	4	3
+	2	5
	6	8

Use deines to represent columnar addition *without exchange* pictorially before moving to abstract algorithm.

We add the ones. 3 ones plus 5 ones are equal to 8 ones.

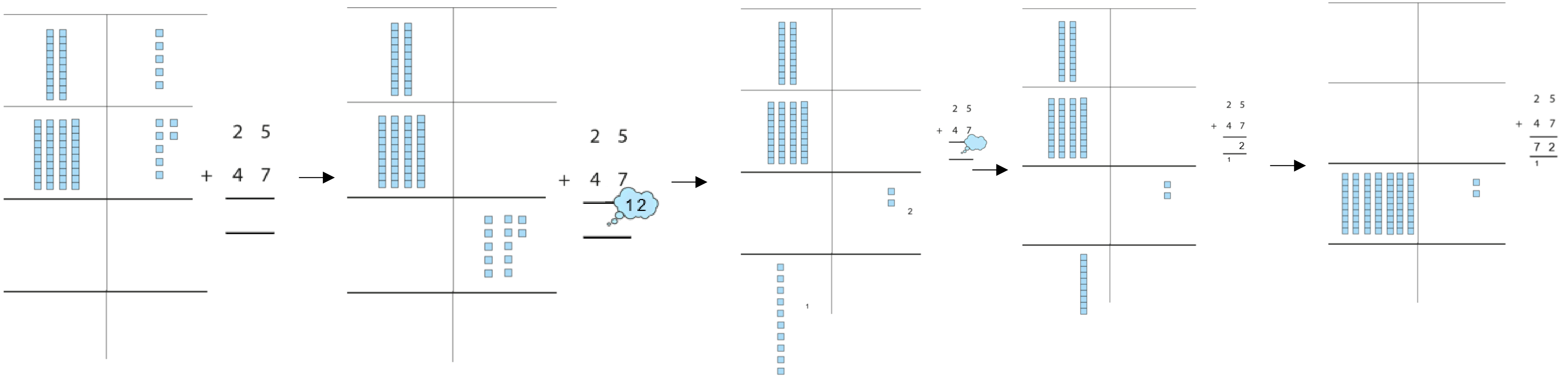
We add the tens. 4 tens plus 2 tens is equal to 6 tens.

Use deines to represent columnar addition *with exchange* pictorially before moving to abstract algorithm.

5 ones plus 7 ones is equal to 12 ones. I can regroup 12 ones. 12 ones is equal to 1 ten and 2 ones.

2 tens plus 4 tens is equal to 6 tens. We also need to add 1 ten from the regrouping. There are 7 tens altogether.

If a column group is equal to 10 or more we must regroup. 10 ones is equal to 1 ten. 10 tens is equal to 1 hundred.



Addition and Subtraction

Year 3

Columnar Addition and Subtraction

Vocabulary:

Ones Tens Represents Compose Combine Total Deines Plus + Minus -
Equal to = Addition Subtraction Equation Expression Regroup Algorithm

Addend + Addend = Sum

Minuend – Subtraction = Difference

475 + 25

237 + 156

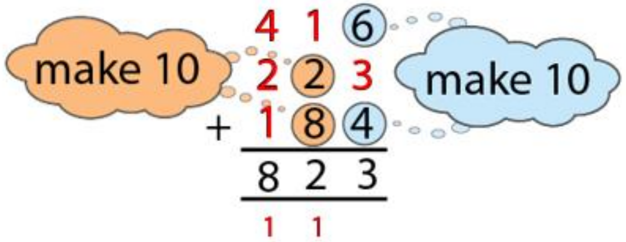
349 + 84

120 + 130

Use column addition	Use mental strategies

Compare expressions which can be calculated using mental or written strategies.

416 + 223 + 184 = 823



Add 3 addends using columnar addition, using a make 10 strategy to support.

Addition and Subtraction

Year 3

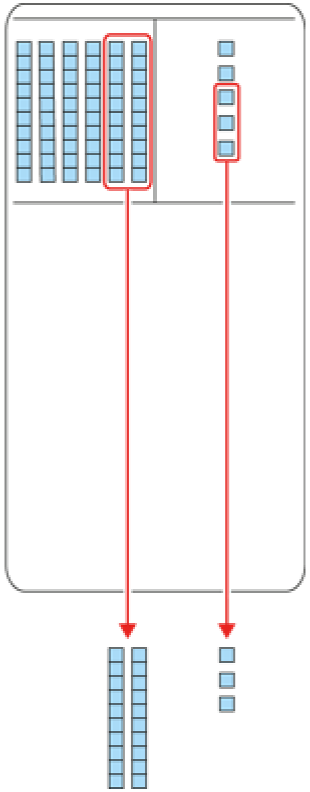
Columnar Addition and Subtraction

Vocabulary:

Ones Tens Represents Compose Combine Total Deines Plus + Minus - Equal to = Addition Subtraction Equation Expression Regroup Algorithm

Addend + Addend = Sum

Minuend – Subtrahend = Difference



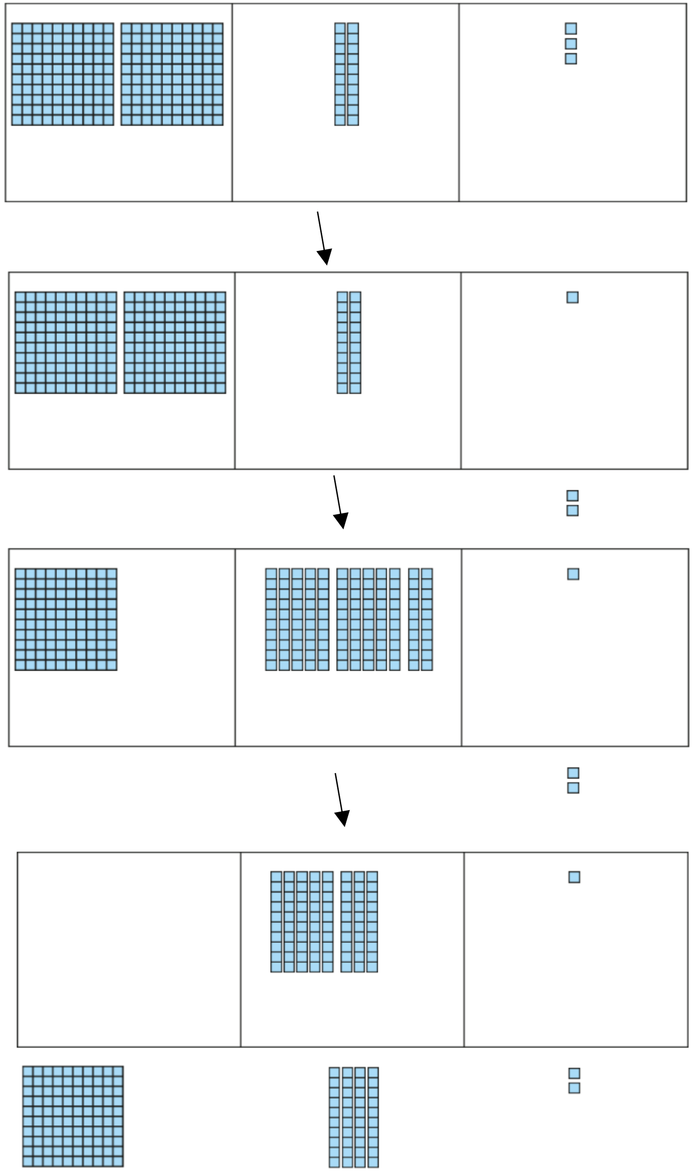
	10s	1s
	6	5
–	2	3
	4	2

Use deines to represent columnar subtraction *without exchange* pictorially before moving to abstract algorithm.

We subtract the ones. 5 ones minus 3 ones is equal to 2 ones.

We subtract the tens. 6 tens minus 2 tens is equal to 4 tens.

$$\begin{array}{rccc} & \cancel{2}^{1} & {}^{1}2 & 3 \\ - & 1 & 4 & 2 \\ \hline & 0 & 8 & 1 \end{array}$$



475 − 358

170 − 140

349 − 101

201 − 198

Use column subtraction	Use mental strategies

Compare expressions which can be calculated using mental or written strategies.

Addition and Subtraction

Year 3

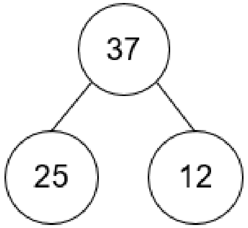
Manipulate the Additive Relationship

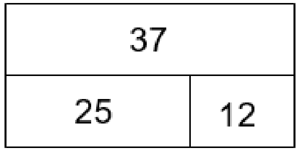
Vocabulary:

Represents Compose Combine Total Deines Plus + Minus - Equal to =
Addition Subtraction Equation Expression Bar Model Part-Part-Whole Model
(Cherry) Whole Part

Addend + Addend = Sum

Minuend – Subtrahend = Difference





25 + 12 = 37
12 + 25 = 37

37 = 25 + 12
37 = 12 + 25

37 – 12 = 25
37 – 25 = 12

25 = 37 – 12
12 = 37 – 25

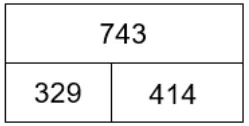
Recognise the different equations that can be recorded based on the part-whole structure.

Addend + addend = sum

Minuend – subtrahend = difference

Part ↓ Part ↓ Whole ↓

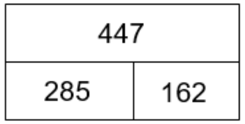
$329 + \boxed{414} = 743$



$743 – 329 = \boxed{414}$

Whole ↓ Part ↓ Part ↓

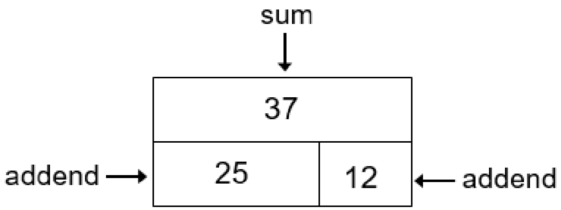
$447 – \boxed{162} = 285$



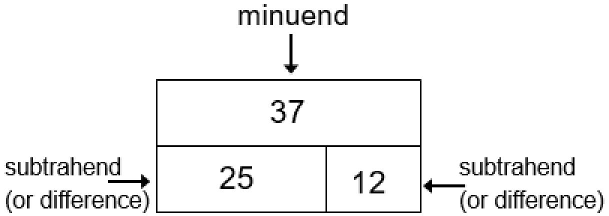
$447 – 285 = \boxed{162}$

Use the part-whole structure to support finding a missing part.

There is a missing part. To find the missing part, we subtract the other part from the whole.



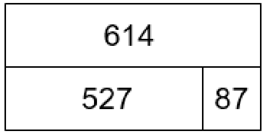
25 + 12 = 37
12 + 25 = 37



37 – 25 = 12
37 – 12 = 25

Whole ↓ Part ↓ Part ↓

$\boxed{614} – 527 = 87$



$527 + 87 = \boxed{614}$

Use the part-whole structure to support finding a missing whole.

There is a missing whole. To find the missing whole, we add the two parts.

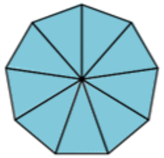
Addition and Subtraction with Fractions

Year 3

Fractions within 1 in the Linear Number System.

Vocabulary:

Fraction Notation Divided Equal Numerator Denominator Whole Parts
Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth
Ninth Tenth One-_____ Linear Number Line Bar Model Vertical Horizontal

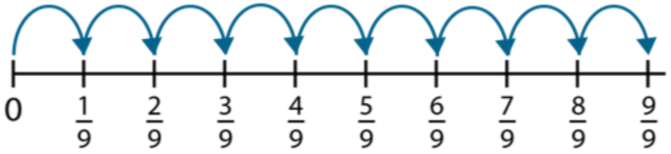


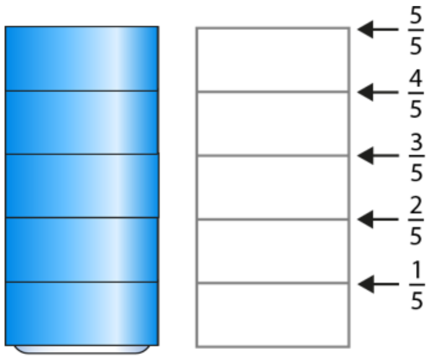
$\frac{9}{9}$

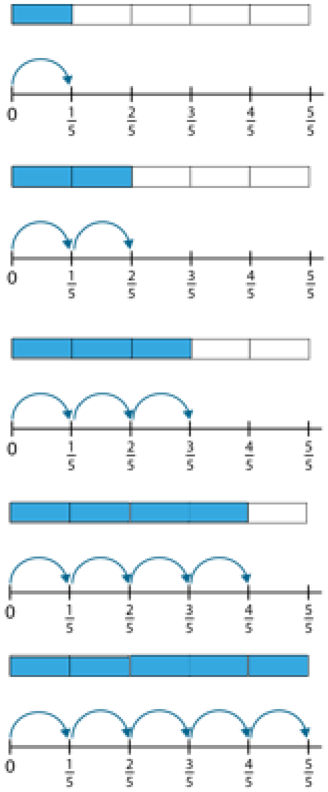
We can represent fractions on both horizontal and vertical number lines.

The whole is divided into ___ equal parts. Each part is ___ of the whole.

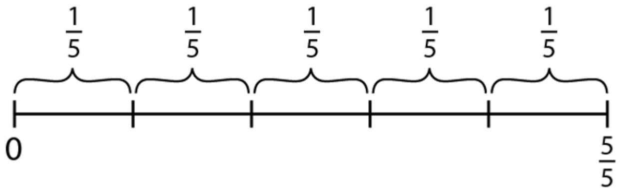
The whole is made up of 9 one-ninths.



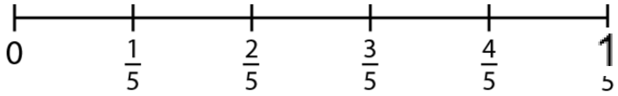




Fractions as part of a whole



Fractions as numbers



Fractions should be seen as part of a whole and as numbers which have their own unique place on a number line.

Generalisation:

When the numerator and denominator are the same, the fraction has a value of 1.

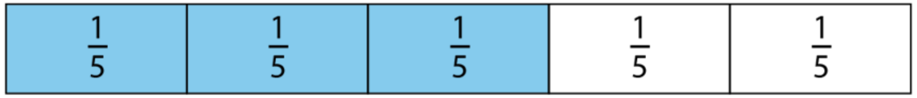
Addition and Subtraction with Fractions

Year 3

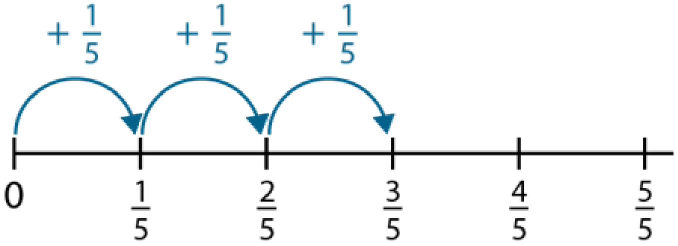
Add and Subtract Fractions within 1

Vocabulary:

Fraction Notation Divided Equal Numerator Denominator Whole Parts Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth Ninth Tenth One-_____ Add Subtract Number line Bar model Equation Expression



$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$



We can add multiples of the unit fraction and record this as an addition equation.

The unit fraction is one-fifth. There are three one-fifths in three-fifths.

Three-fifths is made up of one-fifth, add another one-fifth, and another one-fifth.

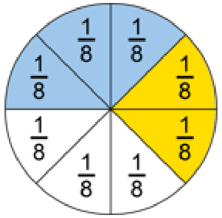
We can use our knowledge of addition and subtraction structures to add/subtract non-unit fractions, recording these as equations.

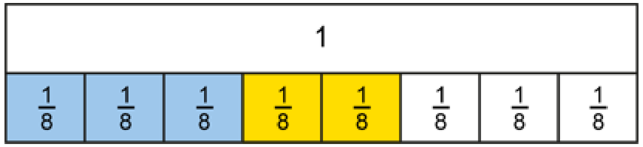
3 one-eighths plus 2 one-eighths is equal to 5 one-eighths.

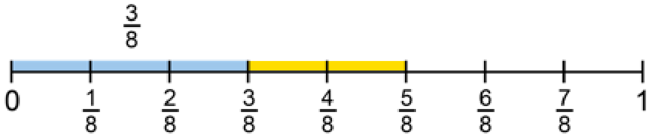
Three-eighths, plus two-eighths is equal to five-eighths.

5 one eighths minus 2 one-eighths is equal to 3 one-eighths.

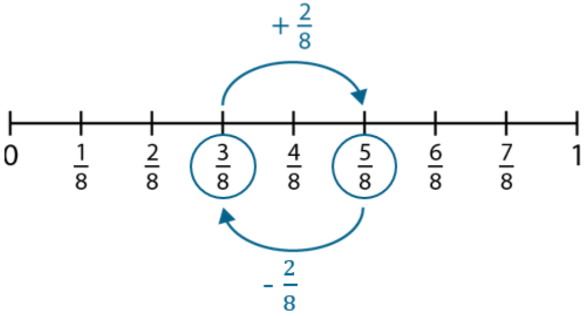
Five-eighths, minus two-eighths is equal to three-eighths.







$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$



$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

Addition and Subtraction with Fractions

Year 3

Add and Subtract Fractions within 1

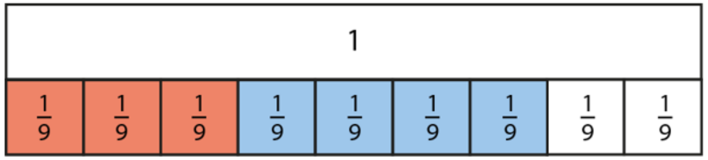
Vocabulary:

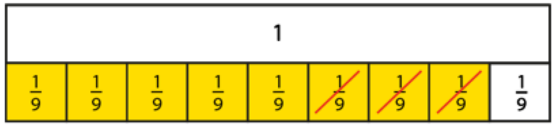
Fraction Notation Divided Equal Numerator Denominator Whole Parts Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth Ninth Tenth One-_____ Add Subtract (Minus) Number line Bar model Equation Expression

We can use one of three methods to represent our understanding of adding and subtracting fractions with the same denominator.

1 – Use a Diagram

***Note – this may best represent an aggregation (adding with) addition structure.**



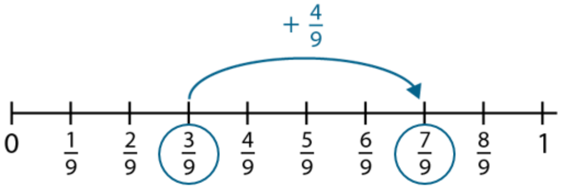


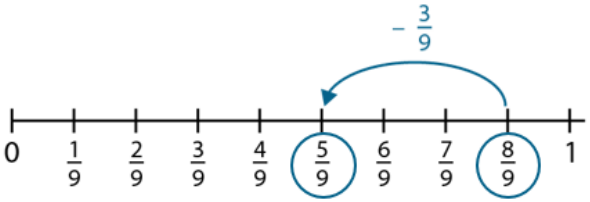
***Note – this may best represent a reductive (take away) subtraction structure.**

***Note – this may best represent a partitioning (separation) subtraction structure.**

2 – Use a Number Line

***Note – this may best represent an augmentation (adding to) addition structure.**





***Note – this may best represent a partitioning (separation) subtraction structure.**

3 – Verbal reasoning

$\frac{3}{9}$ is 3 lots of $\frac{1}{9}$

$\frac{4}{9}$ is 4 lots of $\frac{1}{9}$

I know that $3 + 4 = 7$

So I know that $\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

$\frac{8}{9}$ is 8 lots of $\frac{1}{9}$

$\frac{3}{9}$ is 3 lots of $\frac{1}{9}$

$8 - 3 = 5$

So $\frac{8}{9} - \frac{3}{9}$ is $\frac{5}{9}$

Generalisation:

When adding/subtracting fractions with the same denominator, just add/subtract the numerators.

Addition and Subtraction

Year 4

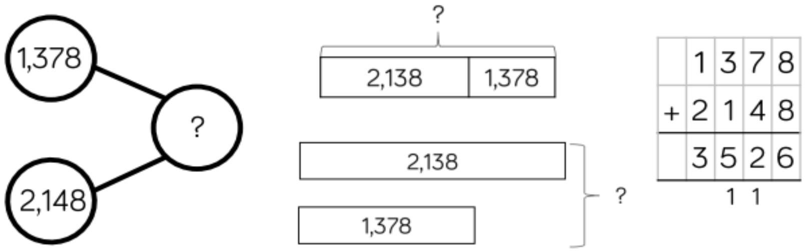
Columnar Addition and Subtraction -Consolidate and extend work from Year 3

Vocabulary:

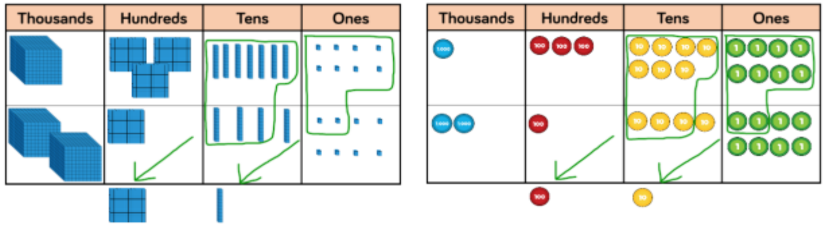
Ones Tens Hundreds Thousands Represents Compose Combine Total Dienes Place Value Counters Plus + Minus - Equal to = Addition Subtraction Equation Expression Regroup Algorithm

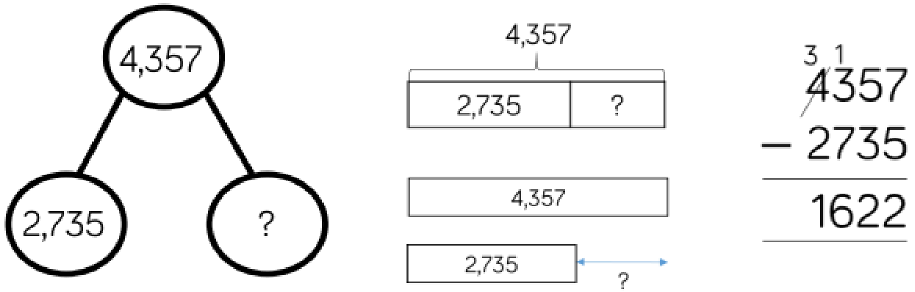
Addend + Addend = Sum

Minuend – Subtrahend = Difference

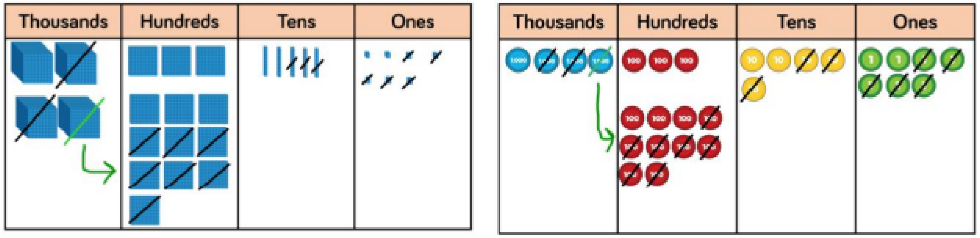


1,378 + 2,148 = 3,526





4,357 – 2,735 = 1,622



Use Deines/Base 10 or Place Value Counters to represent columnar addition and subtraction, first without exchange and then with exchange.

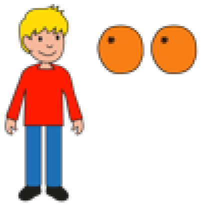
Addition and Subtraction with Fractions

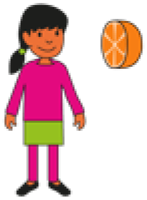
Year 4

Mixed Numbers in the Linear Number System

Vocabulary:

Fraction Notation Divided Equal Numerator Denominator Whole Parts
Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth
Ninth Tenth One-_____ Add Subtract (Minus) Number line Part-Part-Whole
Model Units Previous Next Estimate Intervals



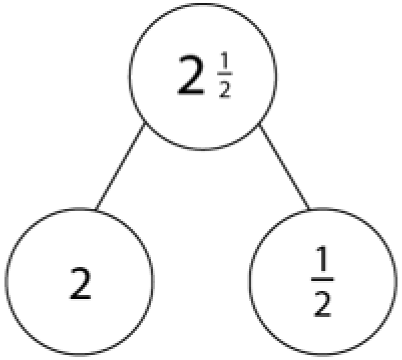


Quantities that are made up of both wholes and parts are called Mixed Numbers.

There are two whole oranges. There is half an orange. There are two and a half oranges altogether.

There are more than two whole oranges.

There are less than three whole oranges.



We can place Mixed Numbers on a number line.

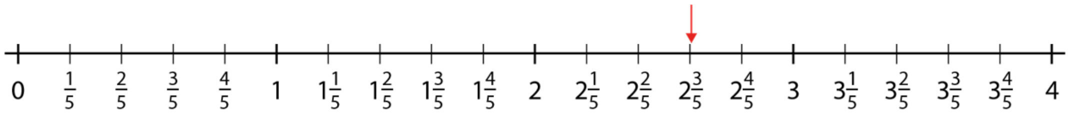
There are ___ parts between zero and one. This means we are counting in units of ___.

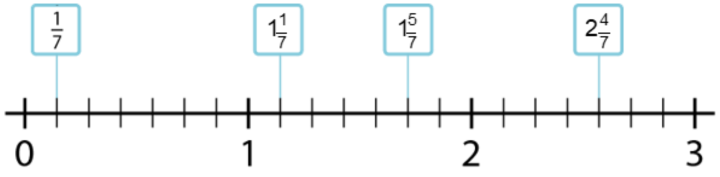
The line is divided into ___ equal parts. This means we are counting in ___s.

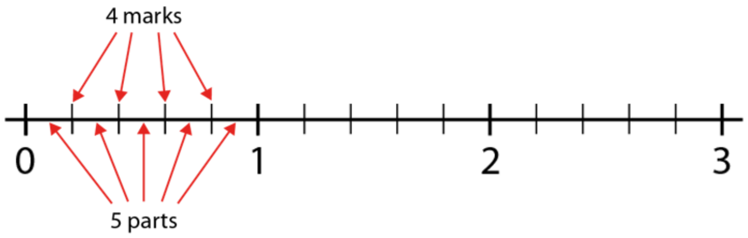
We can use our knowledge of ordering proper fractions to order Mixed Numbers.

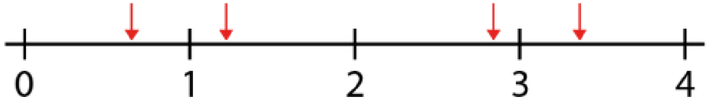
$1\frac{1}{7}$ is between 1 and 2. The previous number is 1. The next number is 2.

We can use our knowledge of placing mixed numbers on a number line to estimate the position of a Mixed Number on a blank number line.









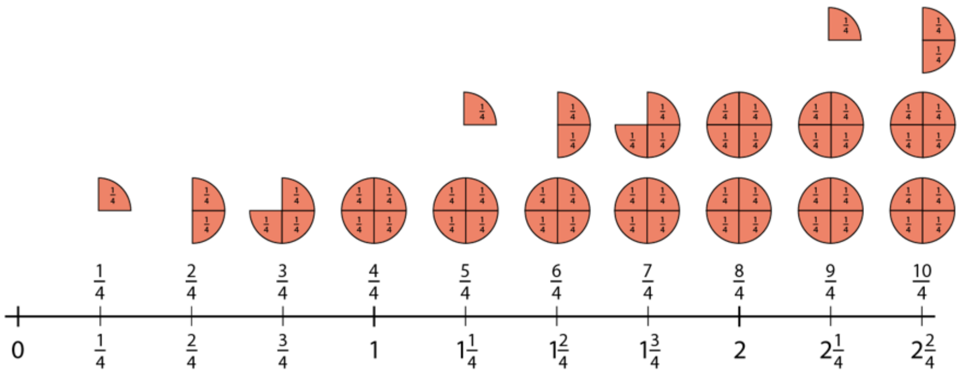
Addition and Subtraction with Fractions

Year 4

Convert between Mixed Numbers and Improper Fractions

Vocabulary:

Fraction Notation Divided Equal Numerator Denominator Whole Parts
Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth
Ninth Tenth One-_____ Number line Part-Part-Whole Model Units Previous
Next Estimate Intervals Convert Improper Fractions Mixed Numbers



We can count in unit fractions over 1 whole and record this as either a Mixed Number or an Improper Fraction.

We can dual count to support this:

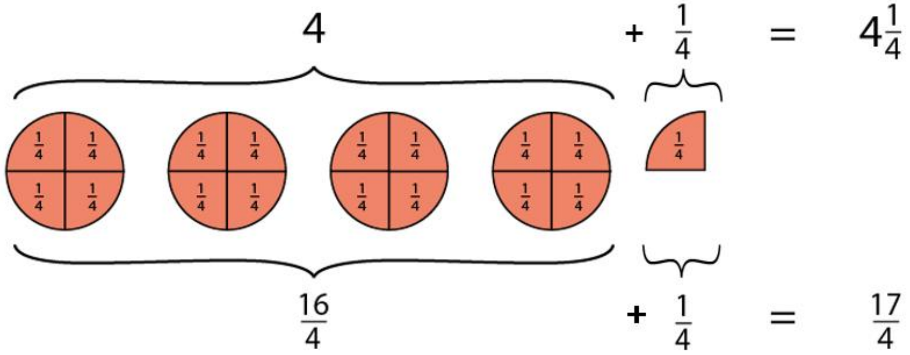
1 quarter, 2 quarter, 3 quarters, 4 quarters, 5 quarters …

1 quarter, 2 quarter, 3 quarters, 1 whole, 1 whole and 1 quarter…

1 group of 4 quarters is 1 whole

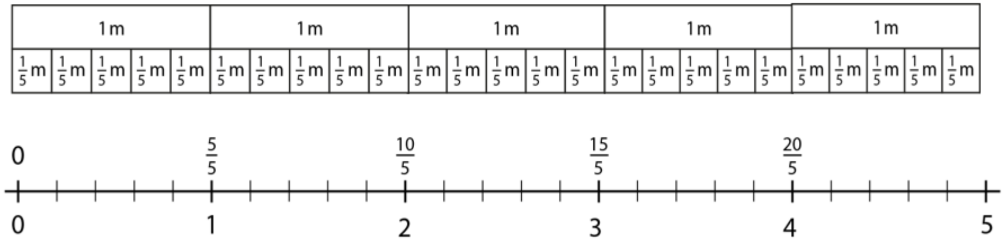
2 groups of 4 quarters in 2 wholes

3 groups of 4 quarters is 3 wholes



There are __ groups of 4 quarters which is __ quarters, and __ more quarters, so that is __ quarters in total.

This counting can be connected to wider contexts including measures.



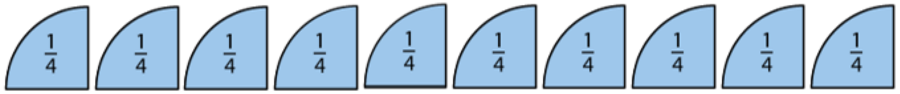
Addition and Subtraction with Fractions

Year 4

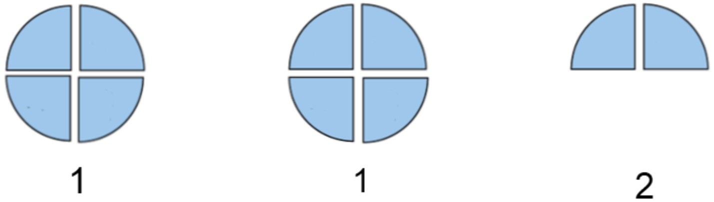
Convert between Mixed Numbers and Improper Fractions

Vocabulary:

Fraction Notation Divided Equal Numerator Denominator Whole Parts Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth Ninth Tenth One-_____ Number line Part-Part-Whole Model Units Previous Next Estimate Intervals Convert Improper Fractions Mixed Numbers



$$\frac{10}{4}$$



$\frac{2}{4}$

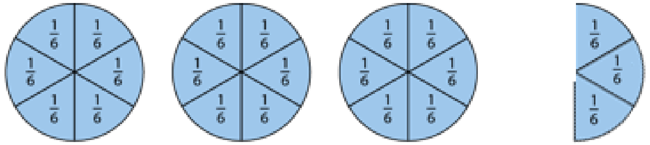
$$\frac{10}{4} = 2\frac{2}{4}$$

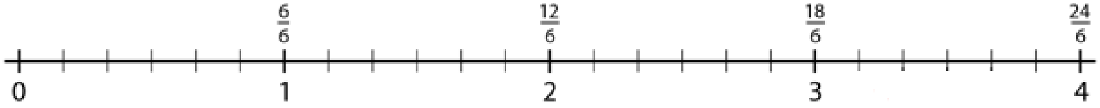
We can convert between Improper Fractions and Mixed Numbers by thinking about the counting unit.

Our unit is quarters so we will be thinking about groups of 4.

There are __ groups of four quarters which is ___-quarters, and ___ more quarters, so that is ___-quarters.

How many groups of 4 quarters in 10 quarters?





We can convert between Improper Fractions and Mixed Numbers by thinking about the counting unit.

Each whole has been divided into ___ equal parts. We have __ of these equal parts. This represents ___ ___s.

This knowledge can be connected to wider contexts including area, quantities, linear and volumes.

Generalise:

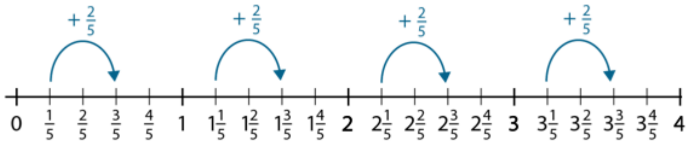
If we multiply the number of wholes by the denominator, we can find the value of the numerator.

Addition and Subtraction with Fractions

Year 4

Add and Subtract Improper Fractions and Mixed Fractions (Same Denominator) (1)

Vocabulary:
Fraction Notation Divided Equal Numerator Denominator Whole Parts
Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth
Ninth Tenth One-_____ Number line Part-Part-Whole Model Units Previous
Next Estimate Intervals Convert Improper Fractions Mixed Numbers Add
Subtract (Minus)

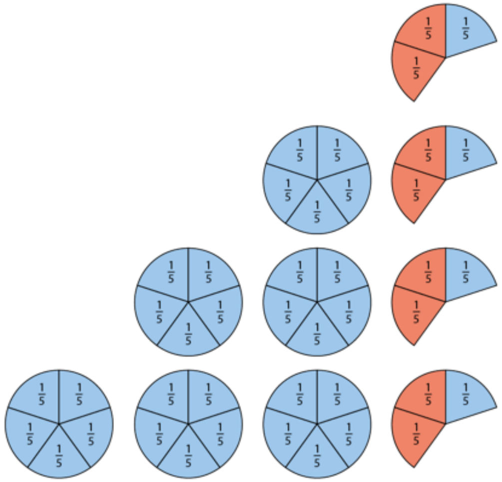


$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

$1\frac{1}{5} + \frac{2}{5} = 1\frac{3}{5}$

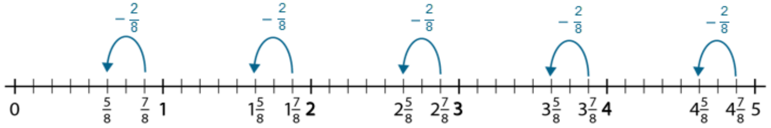
$2\frac{1}{5} + \frac{2}{5} = 2\frac{3}{5}$

$3\frac{1}{5} + \frac{2}{5} = 3\frac{3}{5}$



We can apply our understanding of adding fractions within one with the same denominator to adding a mixed number and fractions within one with the same denominators.

The parts are __ and __. The total, or whole, is __.



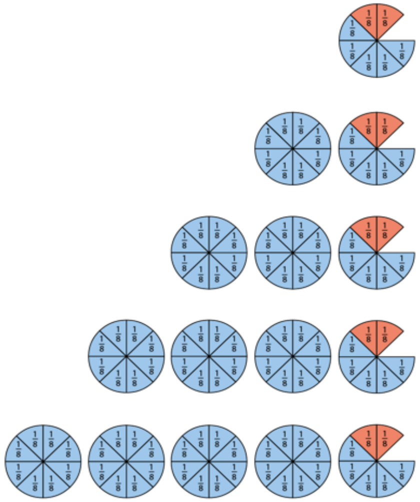
$\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

$1\frac{7}{8} - \frac{2}{8} = 1\frac{5}{8}$

$2\frac{7}{8} - \frac{2}{8} = 2\frac{5}{8}$

$3\frac{7}{8} - \frac{2}{8} = 3\frac{5}{8}$

$4\frac{7}{8} - \frac{2}{8} = \boxed{4\frac{5}{8}}$



We can apply our understanding of subtracting fractions within one with the same denominator to subtract a fraction within one from a mixed number with the same denominators.

The total, or whole, is __. One part is __. The missing part is ___.

$\frac{1}{10} + 3\frac{2}{10} + 4 + \frac{1}{10}$

When adding combined mixed numbers and fractions within one, we combine the parts and then combine the wholes.

The parts are __ and __. The total, or whole, is __.

$\frac{1}{10}$ + $3\frac{2}{10}$ + 4 + $\frac{1}{10}$

$7\frac{4}{10}$

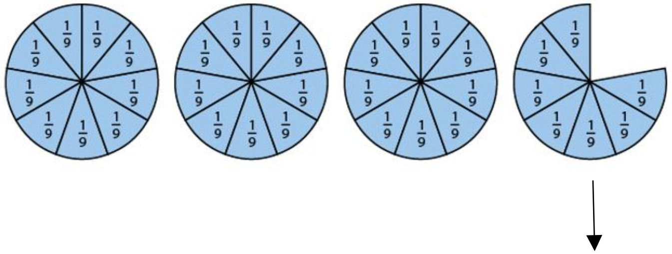
Addition and Subtraction with Fractions
Year 4
Add and Subtract Improper Fractions and Mixed Fractions (Same Denominator) (2)

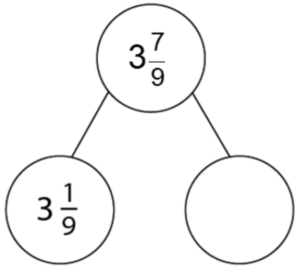
Vocabulary:
Fraction Notation Divided Equal Numerator Denominator Whole Parts
Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth
Ninth Tenth One-_____ Number line Part-Part-Whole Model Units Previous
Next Estimate Intervals Convert Improper Fractions Mixed Numbers Add
Subtract (Minus)

When subtracting fractions within one from a mixed number, we subtract the fraction to reveal the missing part. We can use a part-whole model to help represent this.

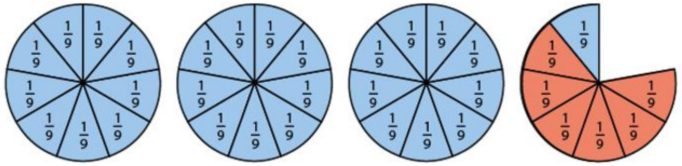
The total, or whole, is __. One part is __. The missing part is ___.

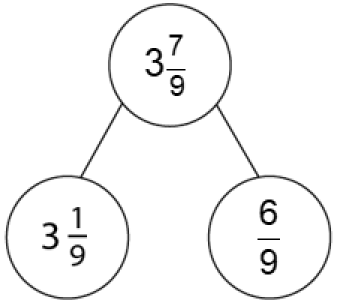
$$3\frac{7}{9} - \boxed{} = 3\frac{1}{9}$$





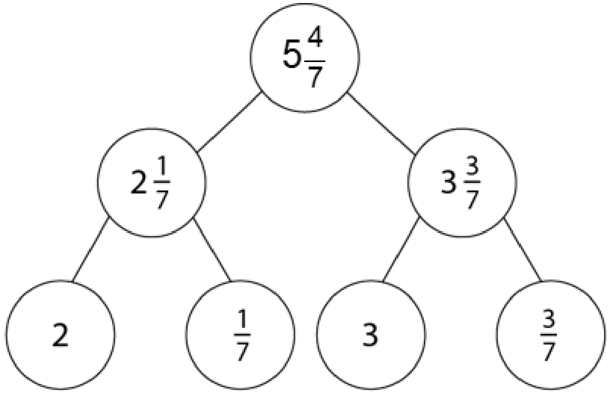
$$3\frac{7}{9} - \boxed{\frac{6}{9}} = 3\frac{1}{9}$$





Representing addition and subtraction of mixed numbers and fractions within one, using a part-whole model can be helpful when problem solving.

The parts are __ and __. The total, or whole, is __.



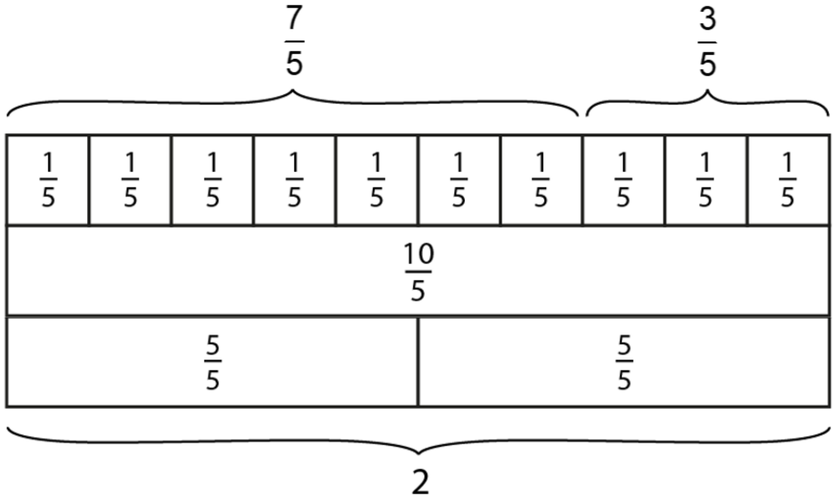
Generalisations:

When adding fractions with the same denominator, just add the numerators.

When subtracting fractions with the same denominator, just subtract the numerators.

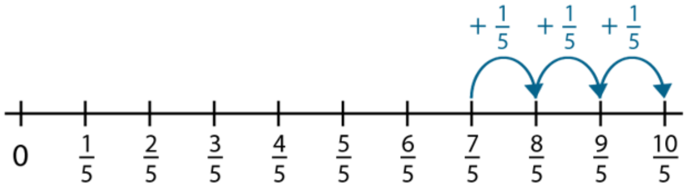
Addition and Subtraction with Fractions
Year 4
Add and Subtract Improper Fractions and Mixed Fractions (Same Denominator) (3)

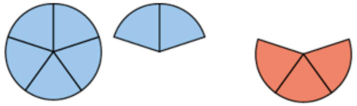
Vocabulary:
Fraction Notation Divided Equal Numerator Denominator Whole Parts Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth Ninth Tenth One-______ Number line Part-Part-Whole Model Units Previous Next Estimate Intervals Convert Improper Fractions Mixed Numbers Add Subtract (Minus)



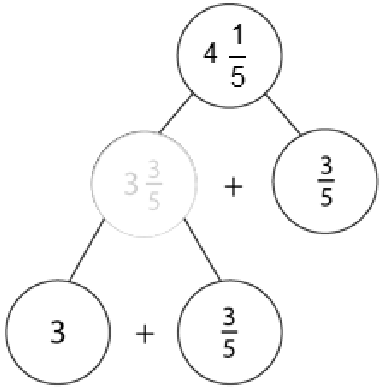
We can apply our understanding of unitising and converting between improper fractions and mixed numbers when adding improper fractions.

7 one-fifths and 3 one-fifths is equal to 10 one-fifths.



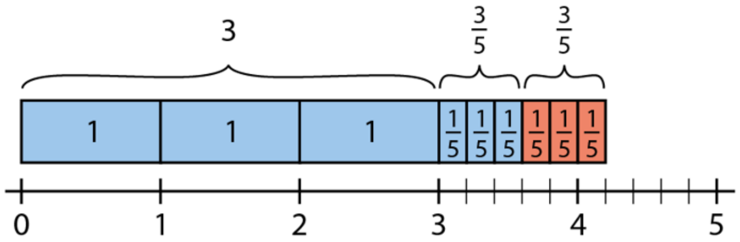


$$\frac{7}{5} + \frac{3}{5} = \frac{10}{5} = 2$$



Partitioning a mixed number and then adding the fractional parts is helpful when adding mixed numbers with fractions within one that result in bridging over a whole.

3 one-fifths and 3 one-fifths is equal to 6 one-fifths. This is equal to one whole and 1 one-fifth.



Addition and Subtraction with Fractions

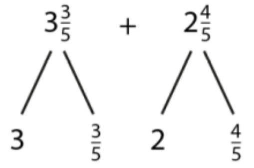
Year 4

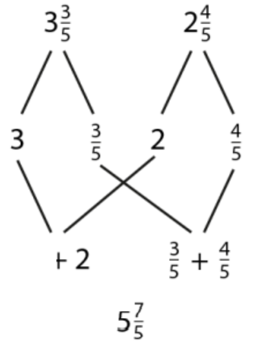
Add and Subtract Improper Fractions and Mixed Fractions (Same Denominator) (4)

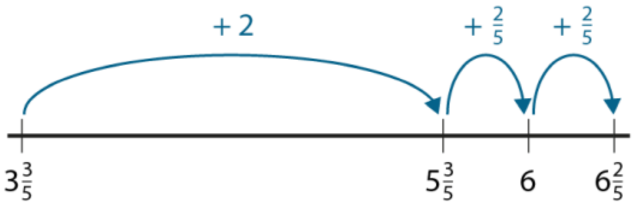
Vocabulary:

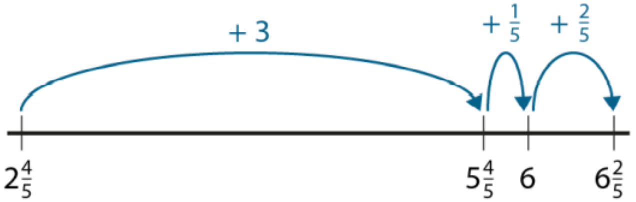
Fraction Notation Divided Equal Numerator Denominator Whole Parts Fraction Bar (Vinculum) Half Third Quarter Fifth Sixth Seventh Eighth Ninth Tenth One-_____ Number line Part-Part-Whole Model Units Previous Next Estimate Intervals Convert Improper Fractions Mixed Numbers Add Subtract (Minus) Aggregation Augmentation Reduction Partitioning Difference

$$3\frac{3}{5}+2\frac{4}{5}=$$

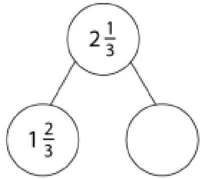


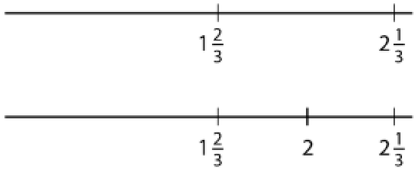






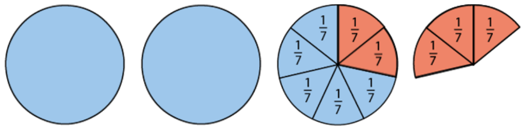
When adding two mixed numbers which bridge a whole, we can apply either a counting on (augmentation) or counting all (aggregation) strategy.

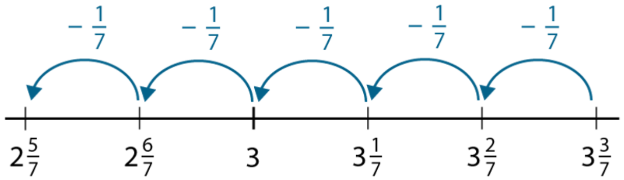




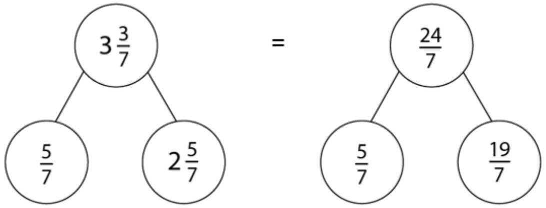
We can also subtract a fraction from a mixed number with the same denominator using our understanding of subtraction as finding the difference.

$3\frac{3}{7}-\frac{5}{7}$

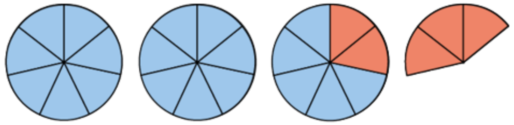




We can subtract a fraction from a mixed number with the same denominator using our awareness of converting between mixed numbers and improper fractions.



$3\frac{3}{7}-\frac{5}{7}=2\frac{5}{7}$



Addition and Subtraction

Year 5

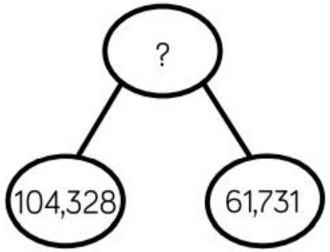
Columnar Addition and Subtraction -Add and subtract numbers with more than 4 digits, first with no exchange and then with exchange.

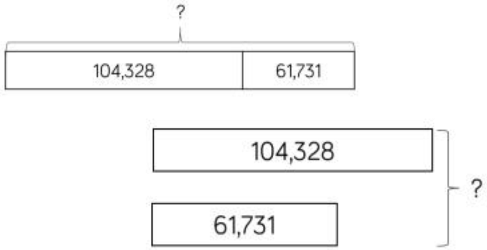
Vocabulary:

Ones Tens Hundreds Thousands Ten thousands Hundred thousands Represents Compose Combine Total Deines Plus + Minus - Equal to = Addition Subtraction Equation Expression Regroup Algorithm

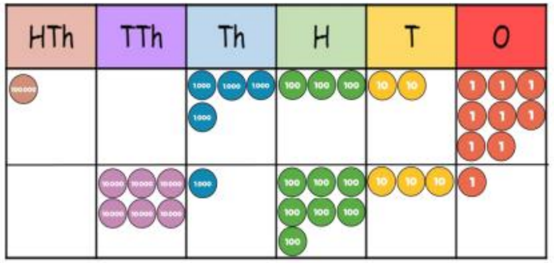
Addend + Addend = Sum

Minuend – Subtrahend = Difference

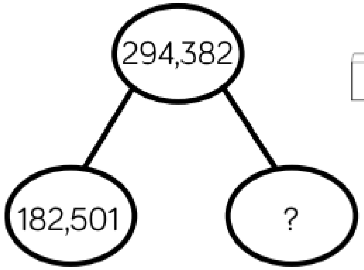


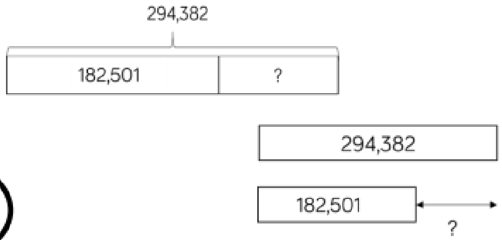


$$104{,}328 + 61{,}731 = 166{,}059$$

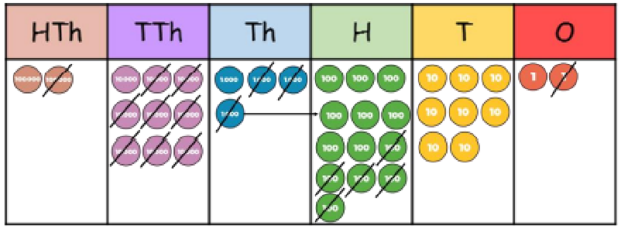


1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			





$$294{,}382 - 182{,}501 = 111{,}881$$



	2	9	$\overset{3}{\cancel{4}}$	$^{1}3$	8	2
–	1	8	2	5	0	1
	1	1	1	8	8	1

Use Place Value Counters to represent columnar addition *with exchange* pictorially before moving to abstract algorithm.

5 ones plus 7 ones is equal to 12 ones. I can regroup 12 ones. 12 ones is equal to 1 ten and 2 ones.

2 tens plus 4 tens is equal to 6 tens. We also need to add 1 ten from the regrouping. There are 7 tens altogether.

If a column group is equal to 10 or more we must regroup. 10 ones is equal to 1 ten. 10 tens is equal to 1 hundred, 10 hundreds is equal to 1 thousand.

Use Place Value Counters to represent columnar subtraction *without exchange and then exchange* pictorially before moving to abstract algorithm.

We subtract the ones. 5 ones minus 3 ones is equal to 2 ones.

We subtract the tens. 6 tens minus 2 tens is equal to 4 tens.

We subtract the hundreds….

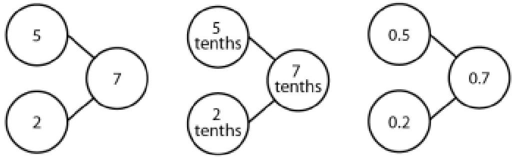
Addition and Subtraction

Year 5

Know facts and strategies, including column algorithms are applied to numbers with tenths.

Vocabulary:
Ones Tens Tenths Whole Represents Compose Combine Total Dienes Place Value Counters Plus + Minus - Equal to = Addition Subtraction Equation Expression Regroup Algorithm Decimal Point
Addend + Addend = Sum
Minuend – Subtraction = Difference

Part–part–whole models:

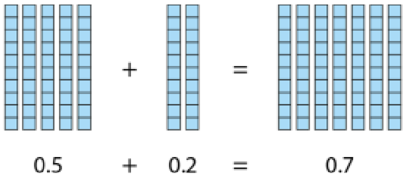


- *'Five tenths plus two tenths is equal to seven tenths.'*
- *'Seven tenths minus two tenths is equal to five tenths.'*

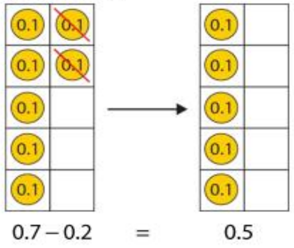
0.5 + 0.2 = 0.7

0.7 – 0.2 = 0.5

Dienes – example addition calculation:

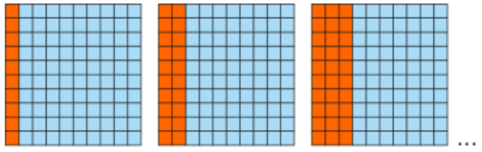


Tens frames and place-value counters – example subtraction calculation:

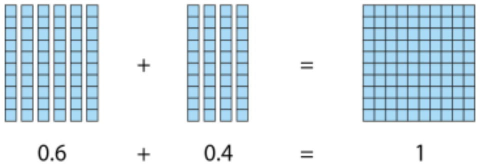


Children apply existing number facts, strategies and methods to numbers with tenths.

Pictorial Dienes representation:

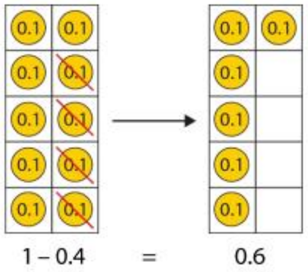


Dienes – example addition calculation:



'Six tenths plus four tenths is equal to ten tenths, which is equal to one.'

Tens frames and place-value counters – example subtraction calculation:

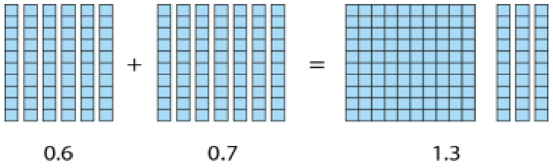


'One is equal to ten tenths; ten tenths minus four tenth is equal to six tenths.'

Continue to use the language of unitising:

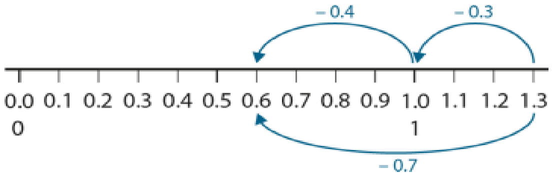
- ***'____ tenths plus ____ tenths is equal to ten tenths, which is equal to one.'***
- ***'One is equal to ten tenths; ten tenths minus ____ tenths is equal to ____ tenths.'***

Dienes – example addition calculation:



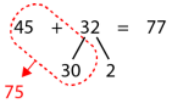
'Six tenths plus seven tenths is equal to thirteen tenths, which is equal to one-point-three.'

Number line – example subtraction calculation:



1.3 – 0.7 = 0.6

Combining known strategies with unitising in tenths:



45 + 32 = 77

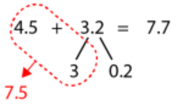
4.5 + 3.2 = 45 tenths + 32 tenths
= 77 tenths
= 7.7

'Complete the calculations.'

$$\begin{array}{r} 3\,.\,2 \\ +\ 5\,.\,7 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\,.\,5 \\ +\ 3\,.\,9 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 6\,.\,5 \\ -\ 2\,.\,3 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\,.\,3 \\ -\ 1\,.\,7 \\ \hline \\ \hline \end{array}$$

Applying known strategies – partitioning numbers with tenths:



Extend column addition to examples where the addends have different number of digits, and subtraction where the minuend has more digits than the subtrahend.

Addition and Subtraction

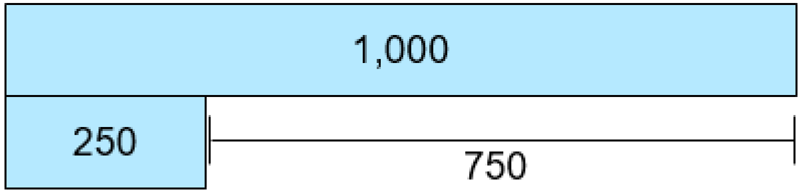
Year 6

Quantify additive and multiplicative relationships

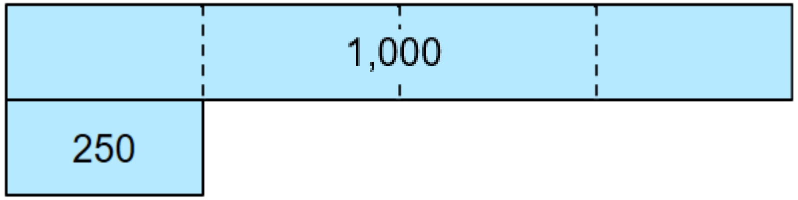
Vocabulary:

Additive Multiplicative Relationship Represents Compose Combine Total
More than Less than Plus + Minus - Equal to = Addition Subtraction Divide ÷
Multiply x One-_____ of Equation Expression Bar Model Whole Part
Difference Multiplier Unknown Sequence

Addend + Addend = Sum

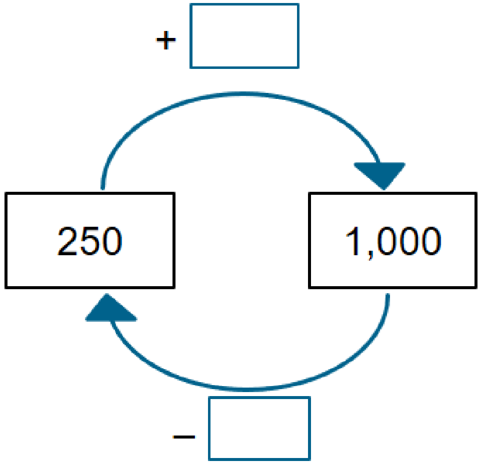


250 + 750 = 1,000 1,000 – 750 = 250



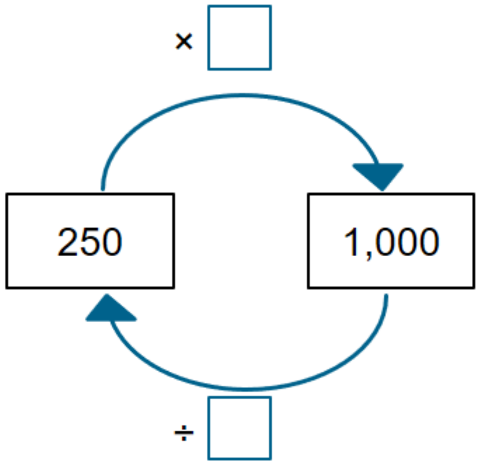
250 x 4 = 1,000 1000 ÷ 4 = 250

The relationship between two numbers can be expressed both additively and multiplicatively.



1000 is ___ more than 250.

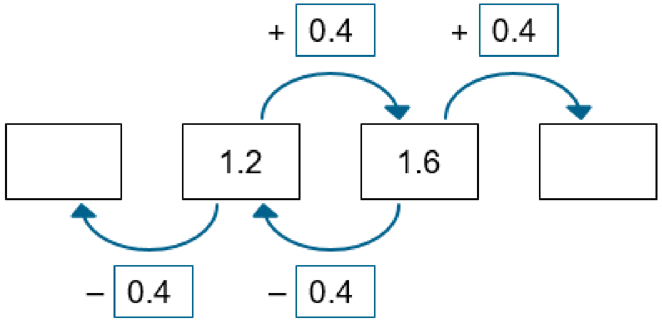
250 is ____ less than 1000.



1000 is ___ times the size of 250.

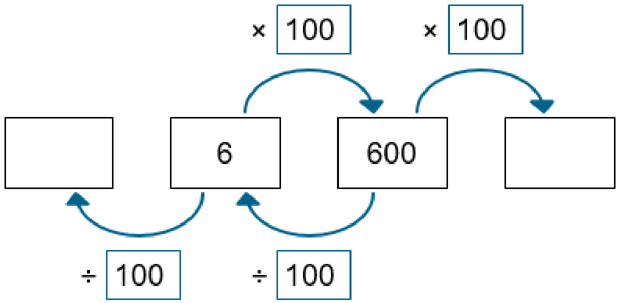
250 is one-_____ of 1000.

To find one-quarter of a number, we divide by 4.



Finding the difference can help calculate the unknown terms in a sequence.

Finding the known multiplier can help calculate the unknown terms in a sequence.



Addition and Subtraction

Year 6

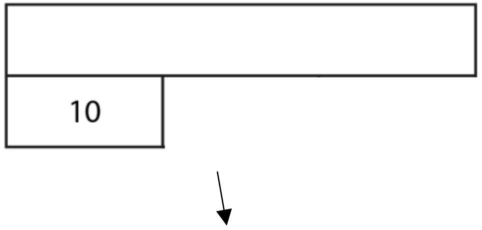
Quantify additive and multiplicative relationships

Vocabulary:

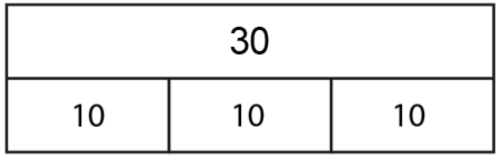
Additive Multiplicative Relationship Represents Compose Combine Total
More than Less than Plus + Minus - Equal to = Addition Subtraction Divide ÷
Multiply x One-_____ of Equation Expression Bar Model Whole Part
Difference Multiplier Unknown Sequence

Addend + Addend = Sum

$\frac{1}{3}$ of ?= 10



$\frac{1}{3}$ of ?= 10



Calculate the unknown whole by recognising how many parts the whole has been divided into.

$\frac{1}{3}$ of 30 = 10

Addition and Subtraction

Year 6

Derive Related Calculations

Vocabulary:

Additive Multiplicative Relationship Represents Equation Unknown Re-arrange Inverse Place Value Properties Commutative Associative Distributive Compensation

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

252 = 3 × 84

2,520 = 30 × ☐

252 = 3 × 84

☐ = 3 × 85

252 = 3 × 84

252 = 3 × 60 + 3 × ☐

625 – 148 = 477

6,250 – 1,480 = ☐

625 – 148 = 477

625 – 70 – ☐ = 477

625 – 148 = 477

625 – 248 = ☐

14.8 + 7.6 = 22.4

1,480 + ☐ = 2,240

14.8 + 7.6 = 22.4

☐ – 7.6 = 14.8

14.8 + 7.6 = 22.4

12.8 + ☐ = 22.4

4,800 ÷ 25 = 192

25 × 192 = ☐

4,800 ÷ 25 = 192

4,800 ÷ 250 = ☐

4,800 ÷ 25 = 192

4,800 ÷ 5 ÷ 5 = ☐

Manipulate an equation to solve another. Pupils could:

- **rearrange the terms;**
- **rewrite using inverse operations;**
- **apply place value;**
- **use the properties of division that correspond to the commutative, associative or distributive property of multiplication;**
- **use the compensation property.**

Additive examples

Multiplicative examples

Addition and Subtraction

Year 6

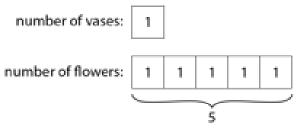
Solve Problems involving Ratio Relationship

Vocabulary:

Additive Multiplicative Relationship Represents Equation Unknown Scale-factor Ratio Ratio Table ___ times the size one-___ the size of Vertical Horizontal

Factor x Factor = Product (Multiplicand x Multiplier = Product)

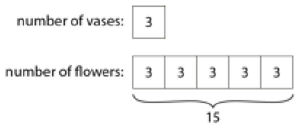
Dividend ÷ Divisor = Quotient



$1 \times \mathbf{5} = 5$

$5 \div \mathbf{5} = 1$

$5 \times \frac{1}{5} = 1$



$3 \times \mathbf{5} = 15$

$15 \div \mathbf{5} = 3$

$15 \times \frac{1}{5} = 3$

Ratio table to compare sets of information.

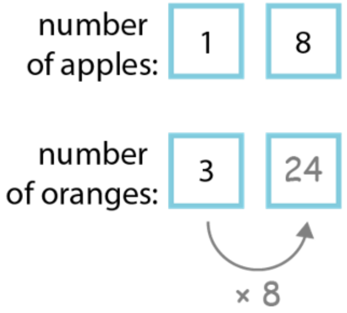
For every ___, there are ___.

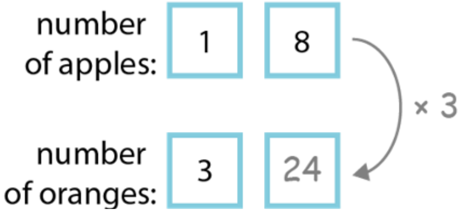
For every 1 litre of petrol, you can drive 7 miles.

For every 7 miles you will drive, you need 1 litre of petrol.

Extend sequences using knowledge of patterns based on ratio table.

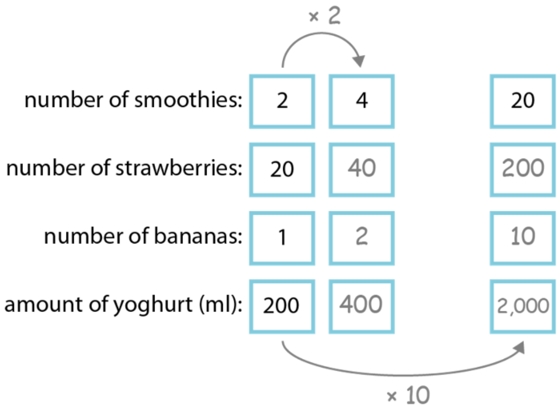
Litres of petrol	1	2	3	4	5	6	7	8	9	10
Miles driven	7	14	21	28	35	42	49	56	63	70





Explore vertical and horizontal relationship between numbers.

For every ___, there are ___.

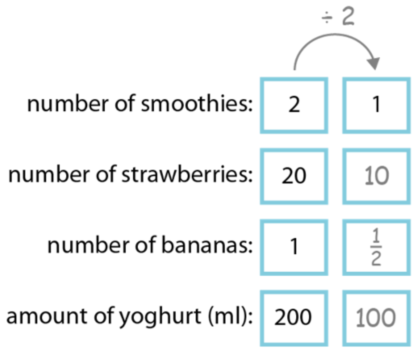


Identify the scale-factor in order to find unknown values.

___ is ___ times the size of ___.

Therefore I must multiply/divide by ___.

___ is one-__ the size of ___.



Addition and Subtraction

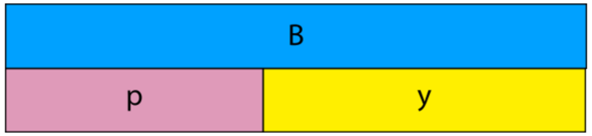
Year 6

Solve Problems with Two Unknowns

Vocabulary:

Additive Multiplicative Relationship Represents Equation Two Unknowns Scale-factor Ratio ___ times the size one-___ the size of Total Bar Model Structure

$$B = \boxed{r} + \boxed{b}$$



$$B = \boxed{p} + \boxed{y}$$

Use Cuisenaire to find 2 bars of total length that are equal to another.

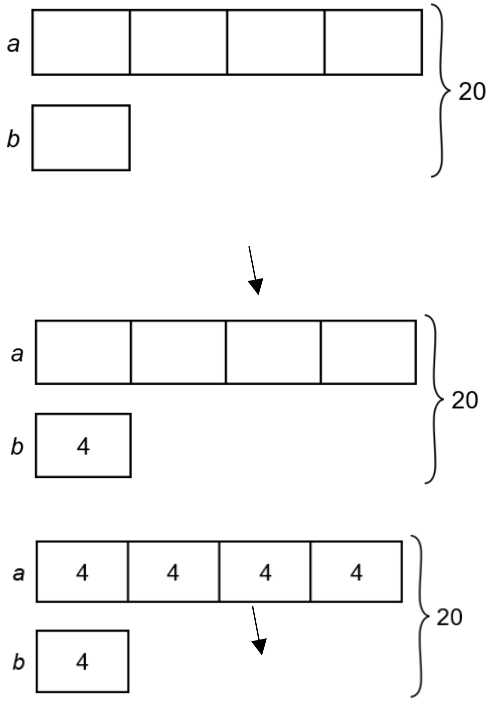
There is more than one solution to the problem.

There can be infinite solutions to a problem.

$$5 \times \boxed{} = 10 \times \boxed{}$$

Solve multiplicative problems with two unknowns when the total is known.

v



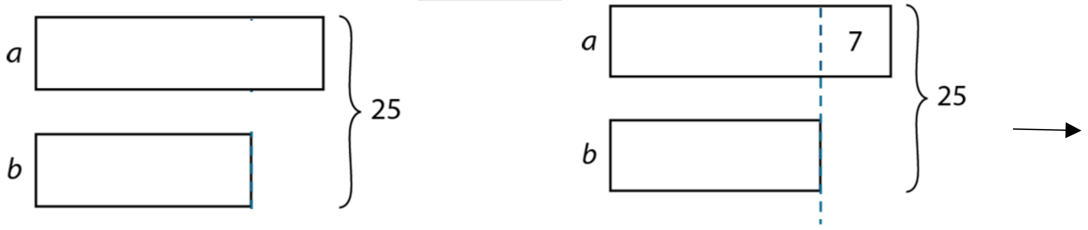
one part = 20 ÷ 5 = 4

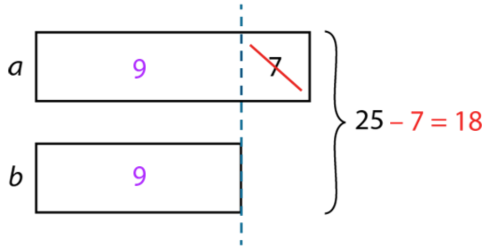
$b = 4$

$a = 4 \times 4 = 16$

The two numbers are 16 and 4.

Solve additive problems with two unknowns when the total is known.





$b = 18 \div 2 = 9$

$a = 9 + 7 = 16$

The two numbers are 9 and 16.

Multiplication and Division

Bar Model

Benefits

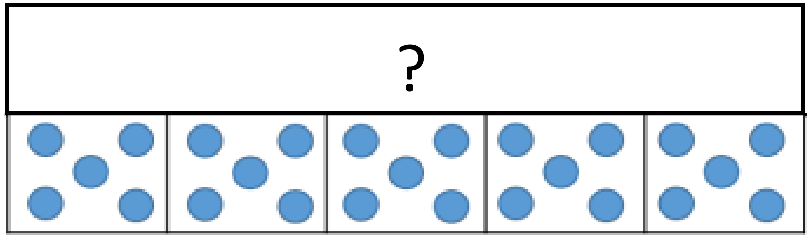
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

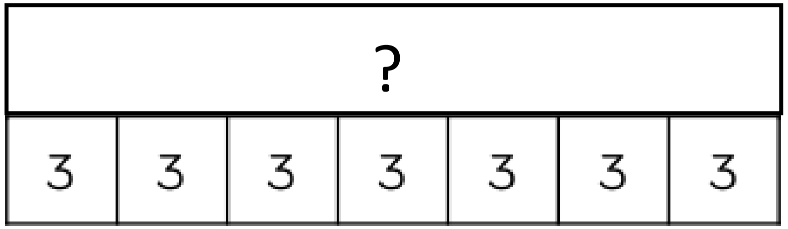
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, eg There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

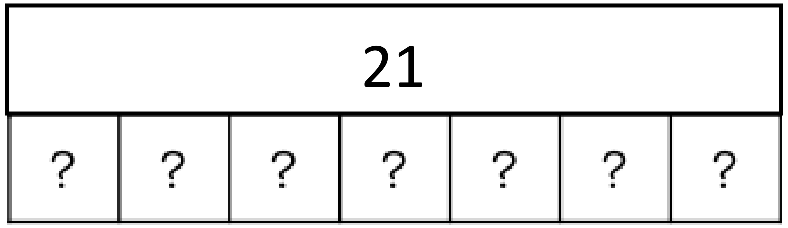


$5 \times 5 = 25$

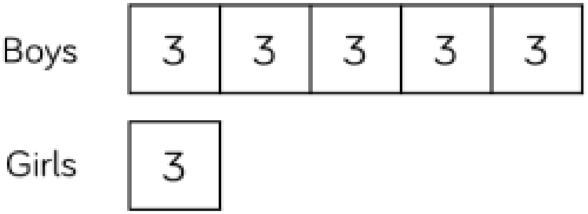


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



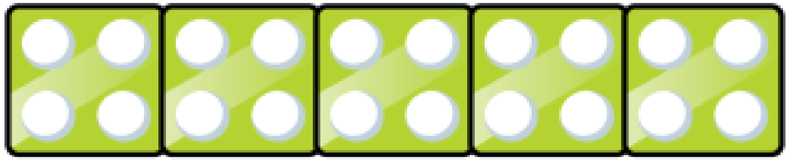
Number Shapes

Benefits

Number shapes support children’s understanding of multiplication as repeated addition.

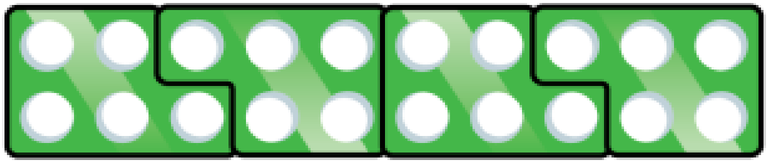
Children can build multiplication in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes on the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication eg odd x odd = even, odd x ever = odd, even x even = even.

When dividing, the number shapes support children’s understanding of division as grouping. Children make the number they are dividing and then place the number shapes they are dividing by over the top of the number to find how many groups of the number there are altogether. Eg There are 6 groups of 3 in 18.



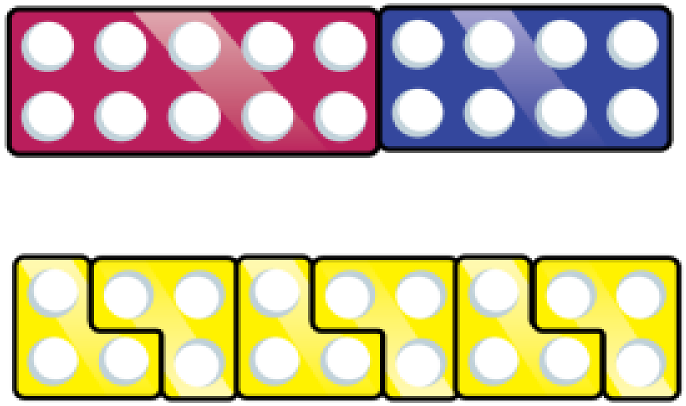
$5 \times 4 = 20$

$4 \times 5 = 20$



$5 \times 4 = 20$

$4 \times 5 = 20$



$18 \div 3 = 6$

Bead Strings

Benefits

Bead strings to 100 support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number eg 4,8,12,16,20.

Children can also use the bead strings to count forwards and backwards in multiples, moving beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by eg 20 divided by 4 – Make 20 and then group the beads into groups of 4. Count how many groups you have made to find the answer. Ensure that children are articulating their counting correctly eg one group of 4, two groups of 4

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

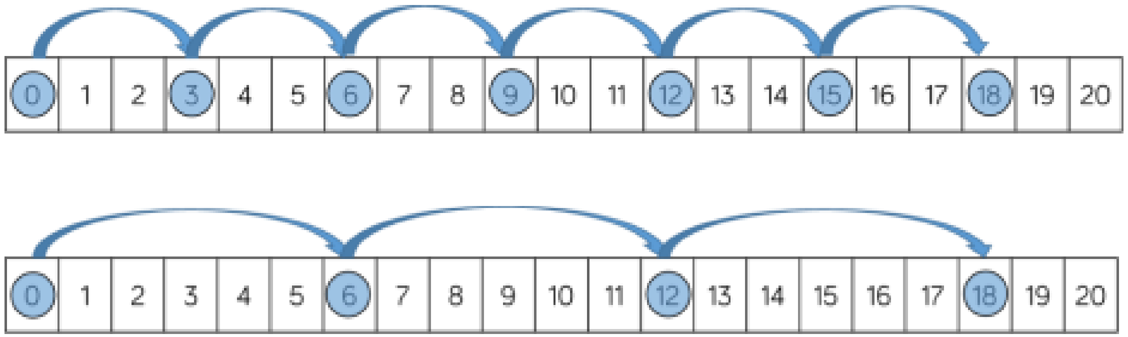
Number Tracks

Benefits

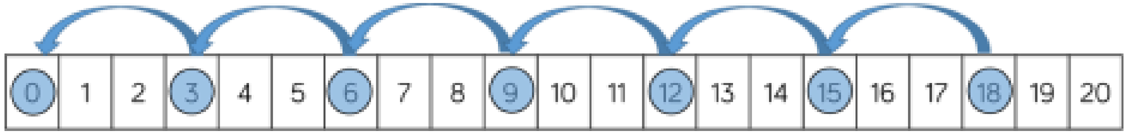
Number tracks support children to count in multiples forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers. When dividing, children place their counter on the number they are dividing and then count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers, they can become less efficient



$$6 \times 3 = 18$$
$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Number Lines (labelled)

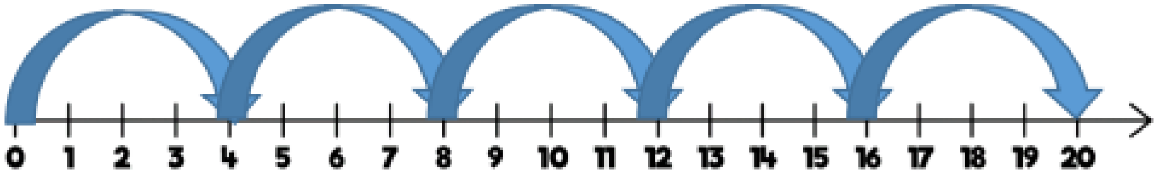
Benefits

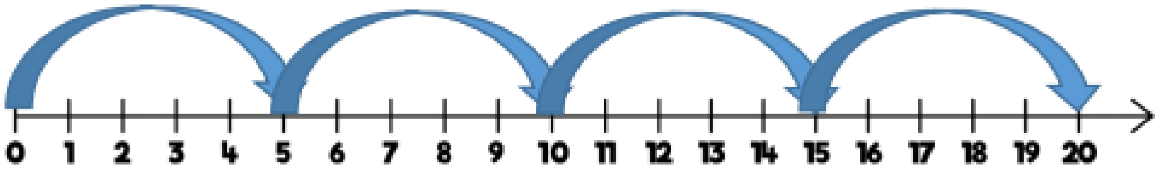
Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and then count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

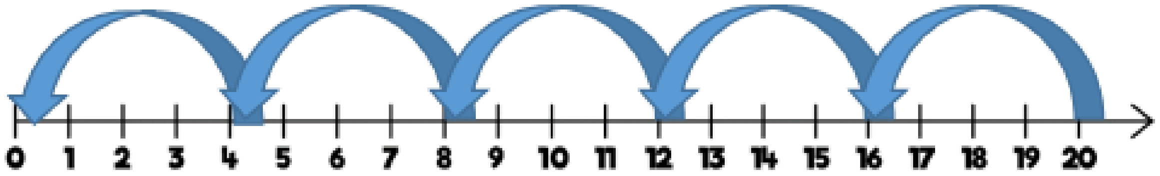
Labelled number lines can be useful with smaller multiples, however they become inefficient as number become larger due to the required size of the number line.





$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

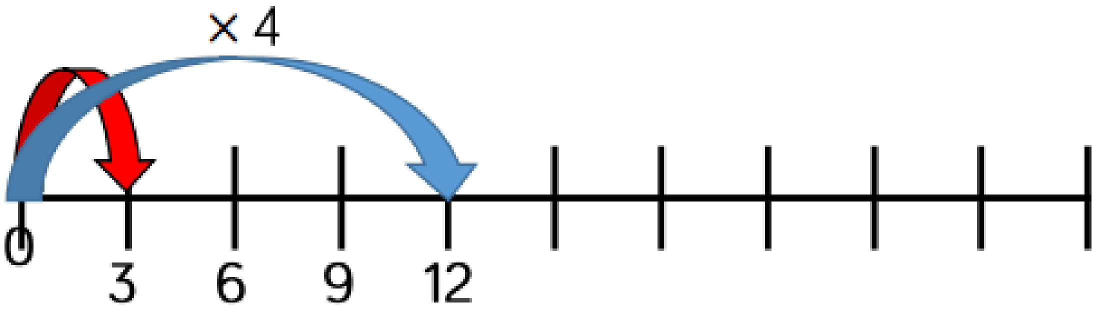
Number Lines (blank)

Benefits

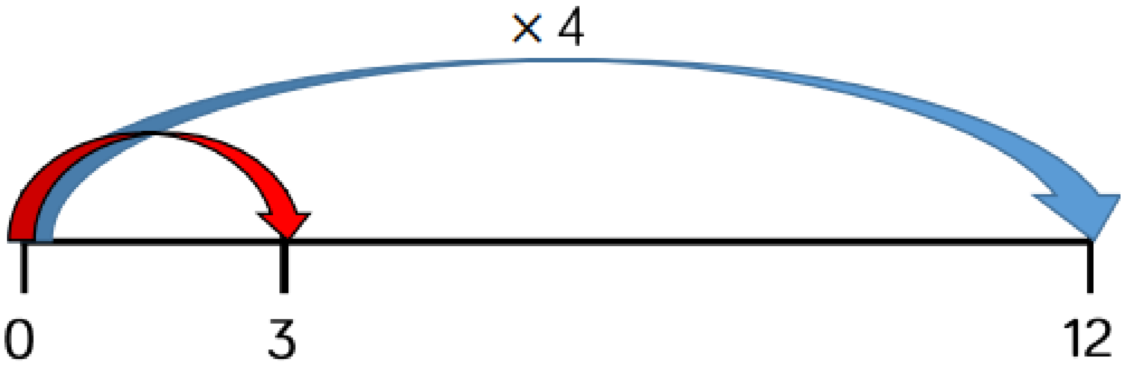
Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with interval can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

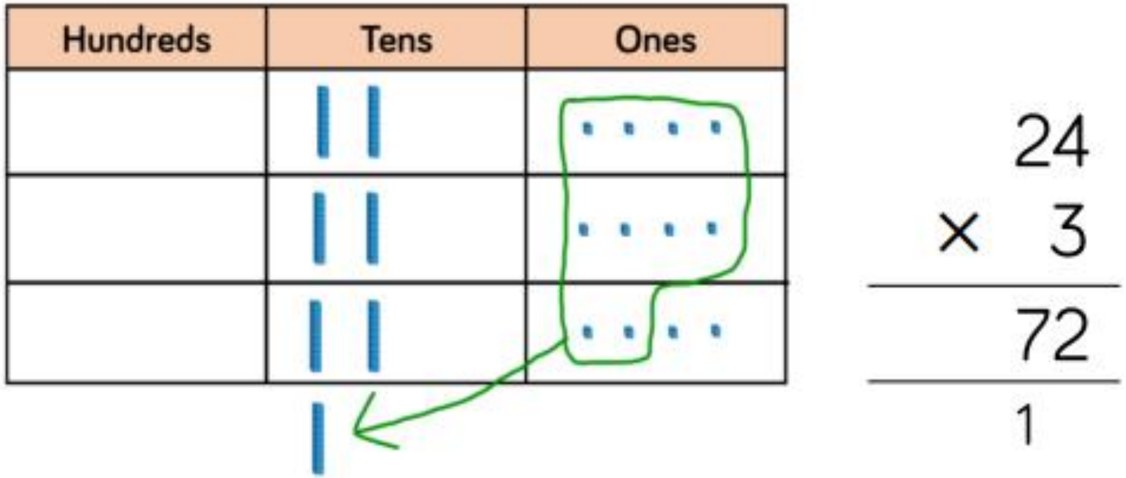
Base 10/Dienes (multiplication)

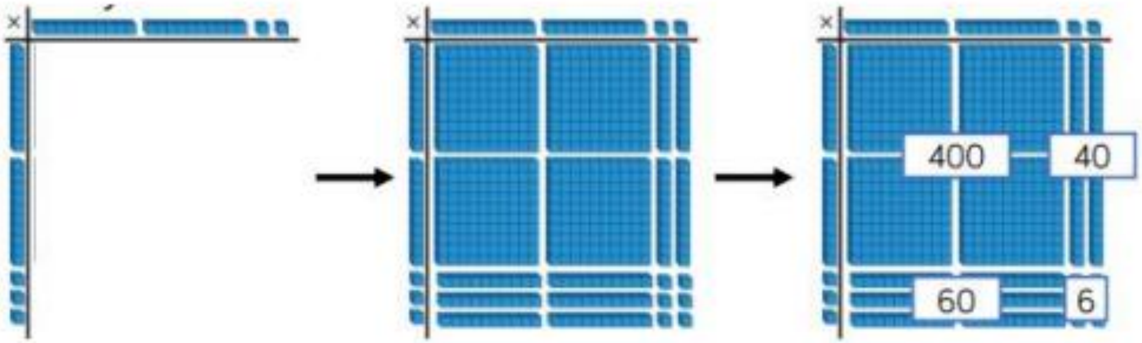
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups become higher, Base 10/dienes become less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then fin the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.





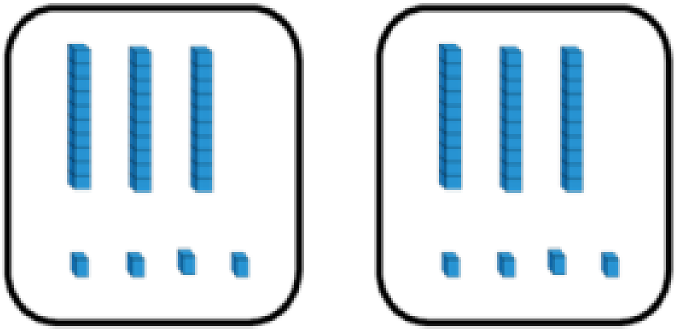
Base 10/Dienes (division)

Benefits

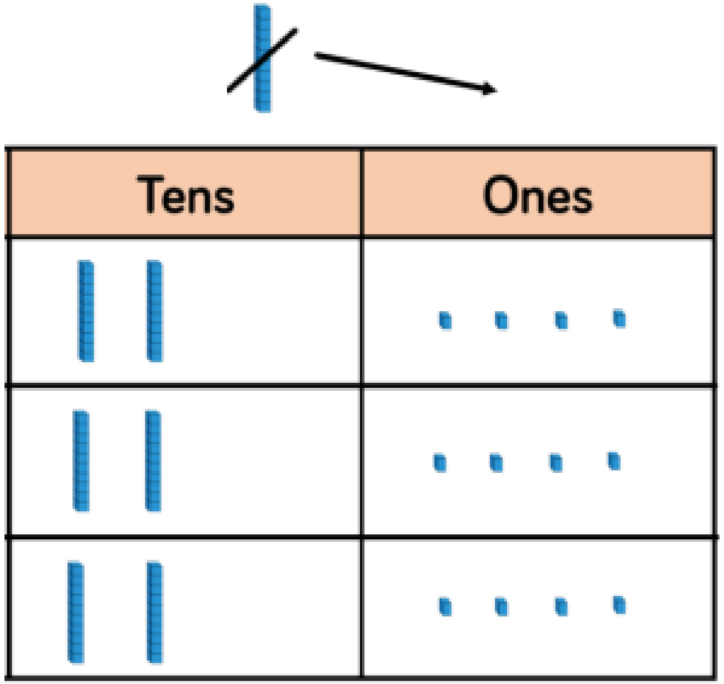
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10 between different groups eg by drawing circles or by rows on a place value grid.

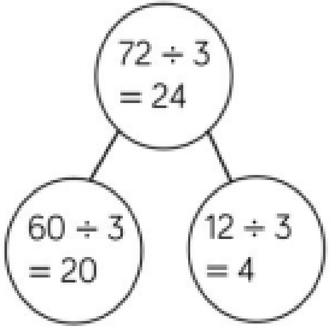
When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange eg one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.



$68 \div 2 = 34$



$72 \div 3 = 24$



Place Value Counters (multiplication)

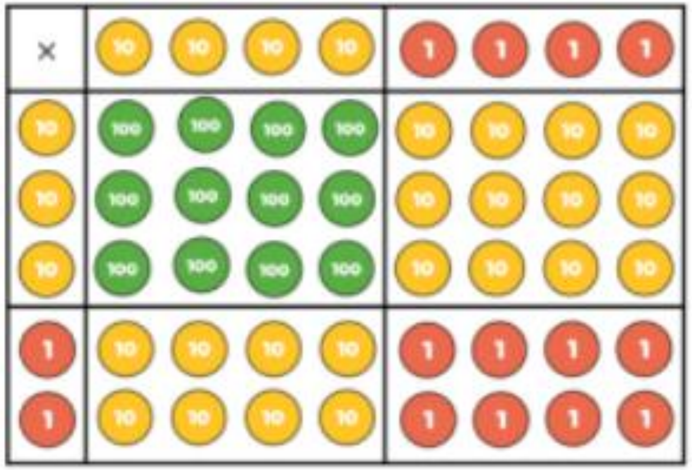
Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

$$\begin{array}{r} 34 \\ \times \quad 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

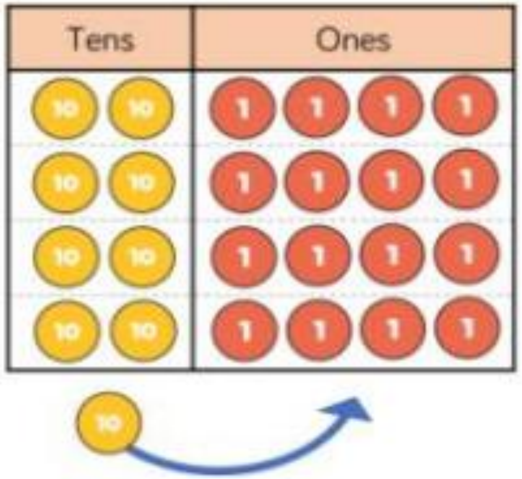
Base 10/Dienes (division)

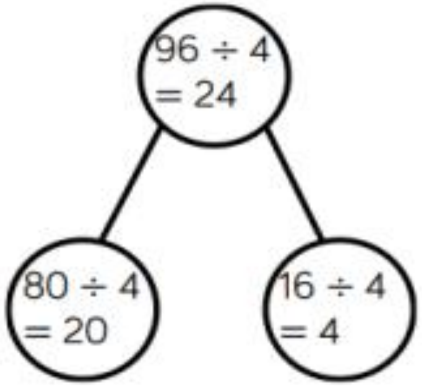
Benefits

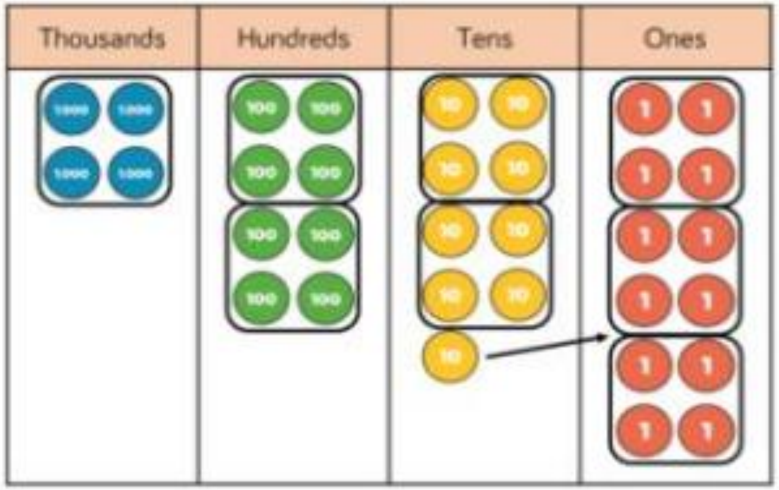
Using place value counters is an effective way to support children’s understanding of division.

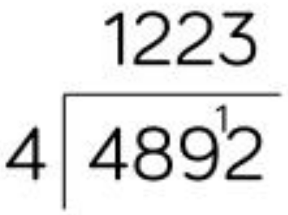
When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. It there are any counters left over once they have been shared, they exchange the counter, eg exchange one ten for then ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children’s understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter eg exchange one hundred for ten tens.









Multiplication and Division

EYFS

Multiplication as Repeated Addition

Vocabulary:

Group Equal Unequal Repeated Addition Multiplication Expression Equation Part Altogether Represents Amount Size

Multiplication and Division

Year 1

Multiplication as Repeated Addition

Vocabulary:

Group Equal Unequal Repeated Addition Multiplication Expression Equation Part Altogether Represents Amount Size

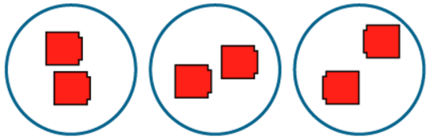
Multiplication and Division

Year 2

Multiplication as Repeated Addition

Vocabary:

Group Equal Unequal Repeated Addition Multiplication Expression Equation
Part Altogether Represents Amount Size



Understand the difference between equal and unequal groups.

The ___ have been grouped.

We can represent equal groups as repeated addition.

There are 3 groups of 5.

5 + 5 + 5

3 × 5

5 + 5 + 5 = 3 × 5

We can represent repeated addition using a multiplication expression.

The 3 represents the number of groups.

The 5 represents the number of eggs in each group.

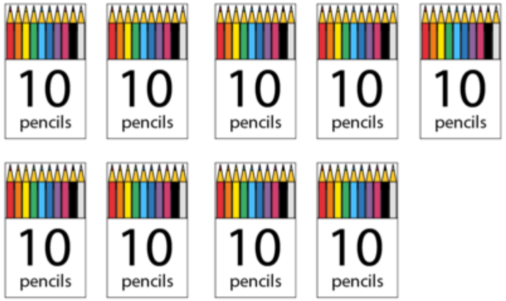
15 represents the total number of eggs.

The ___ represents the number of groups.

The __ represents the number of ___ in each group.

___ represents the total number of ___.

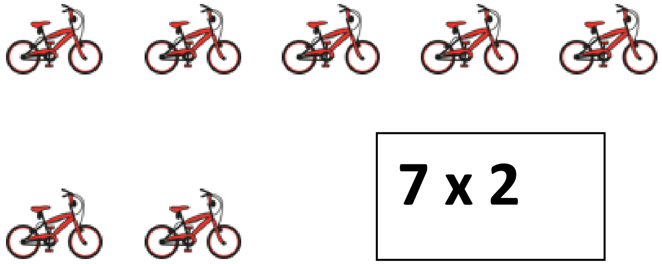
Notice how the representations allow the children to see each of the numbers (i.e. 10



9 × 10

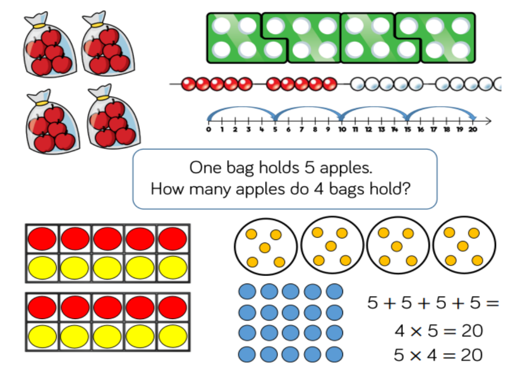
We can skip count in multiples of __ to work out the total amount.

10, 20, 30, 40 … there are 90 pencils altogether.



7 x 2

Appropriate representations are selected to support the calculation.



Multiplication and Division

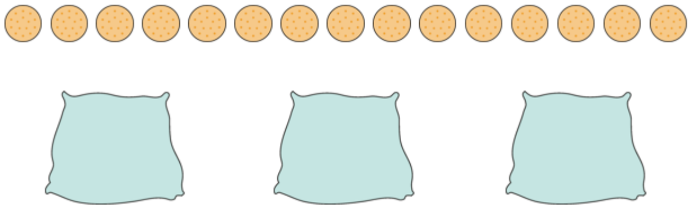
Year 2

Grouping problems: missing factors and division

Vocabulary:

Multiplication   Division   Factor   ‘divided by’   Represents   Skip Counting

Multiplication facts   Groups   Amount   Size



We can solve division problems by finding missing factors.

$\boxed{3} \times 5 = 15$

$15 \div 5 = \boxed{3}$

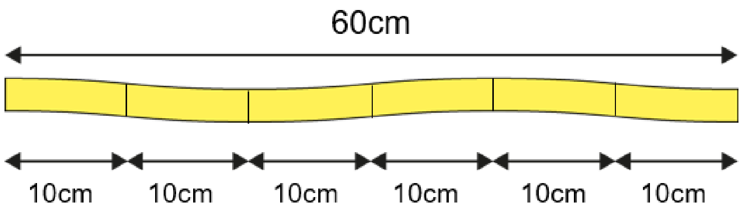
The 15 represents the number of biscuits.

The 5 represents the number of biscuits in each bag (group).

The 3 represents the number of bag (groups).

We can use ÷ to mean ‘divided by’

We can use our knowledge of times tables to help solve division problems.



$\boxed{6} \times 10 = 60$

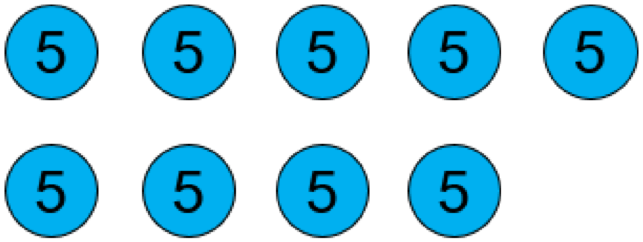
$60 \div 10 = \boxed{6}$

The 60cm represents the length of the ribbon.

The 10 represents the size of each piece.

The 6 represents the number of pieces we can make.

$45 \div 5 = \boxed{9}$



Multiplication and Division

Year 3

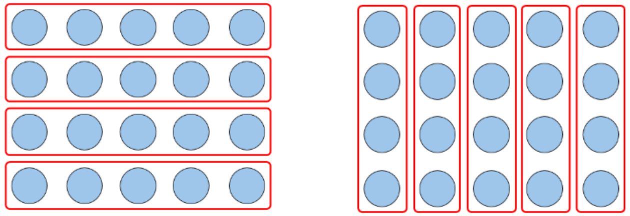
Multiplication and Division Structures

Vocabulary:

Multiplication Division Commutative Grouping (Quotitive) Sharing (Partitive)

'Divided into' 'Divided between' 'Divided by' Equation Factor Product

30	÷	5	=	6
dividend	÷	divisor	=	quotient

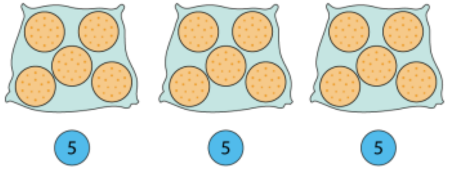


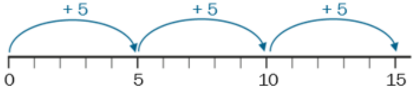
Identify that multiplication is commutative.

4 x 5 = 5 x 4

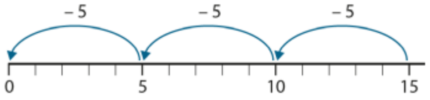
Factor times factor is equal to product.

The order of the factors does not affect the product.

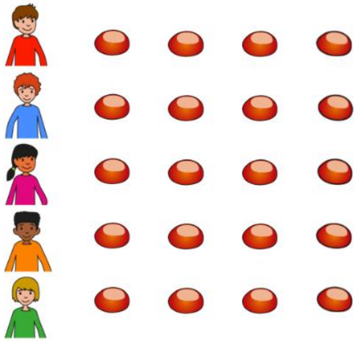


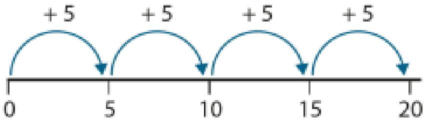


Making groups of

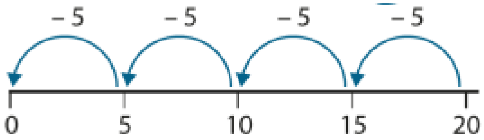


Removing groups of





Receiving a share



Giving a share

Division equations can be used to represent 'grouping' problems.

We can use multiplication facts to find the number of groups.

(Quotitive division)

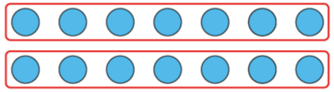
15 divided into groups of 5 is equal to 3.
5 + 5 + 5 = 15
15 – 5- 5 – 5 = 0
15 ÷ 5 = 3

Division equations can be used to represent 'sharing' problems.

We can use multiplication facts to find the size of groups.

(Partitive division)

Four fives are four each.
20 divided between 5 is equal to 4 each.
20 ÷ 5 = 4

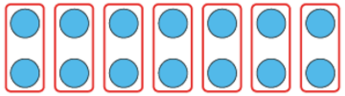


14 ÷ 2 = 7

14	
7	7

The same equation can be represented in both grouping and sharing contexts.

7 times 2 is 14, so 14 ÷ 2 = 7



14 ÷ 2 = 7

14						
2	2	2	2	2	2	2

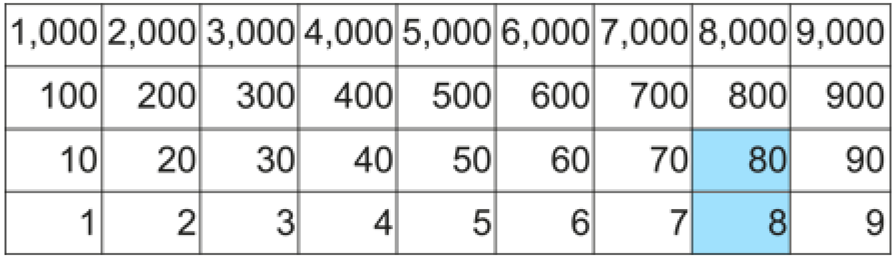
Multiplication and Division

Year 4

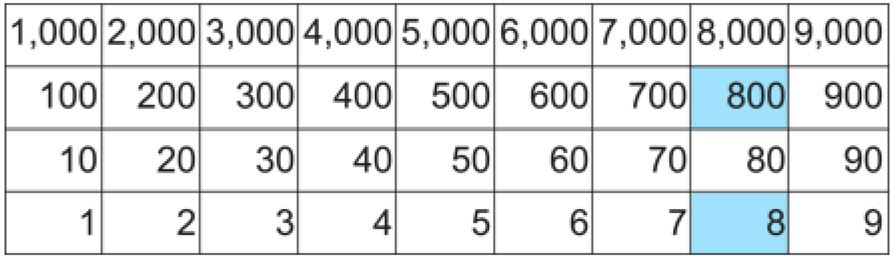
Multiplying and Dividing by 10 and 100

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Develop language in order to multiply and divide by 10 or 100.

80 is ten times bigger than 8.
8 is ten times smaller than 80.
80 is ten times the size of 8
8 is one-tenth the size of 80.

800 is one hundred times bigger than 8.
8 is one hundred times smaller than 800.
800 is on hundred times the size of 8
8 is one-hundredth the size of 80.

8 x 1 = 8
8 x 1 ten – 8 tens
8 x 1 hundred = 8 hundreds

Generalisations

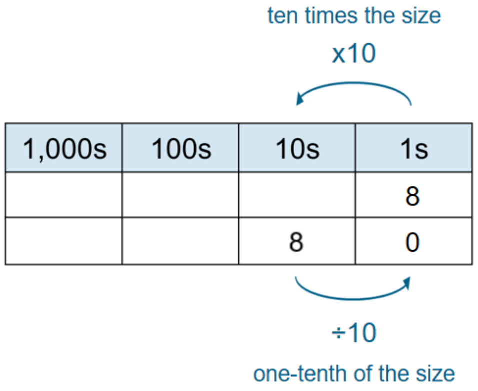
All multiples of 10 have a ones digit of zero.

All multiples of 100 have both a tens and ones digit of zero.

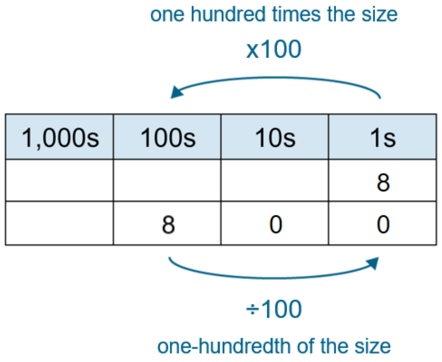
To find the inverse of ___times as many, you divide by ____.

If one factor if made ___ times bigger/smaller then the product will be ten times bigger/smaller

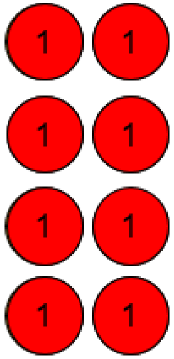
8 made ___ times the size is ___.

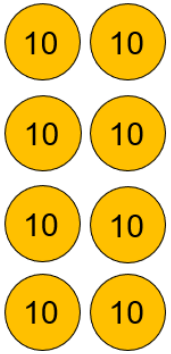


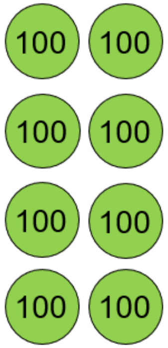
1,000s	100s	10s	1s
			8
		8	0



1,000s	100s	10s	1s
			8
	8	0	0







8 × 1 = 8 8 × 10 = 80 8 × 100 = 800

8 groups of ___ is ___.

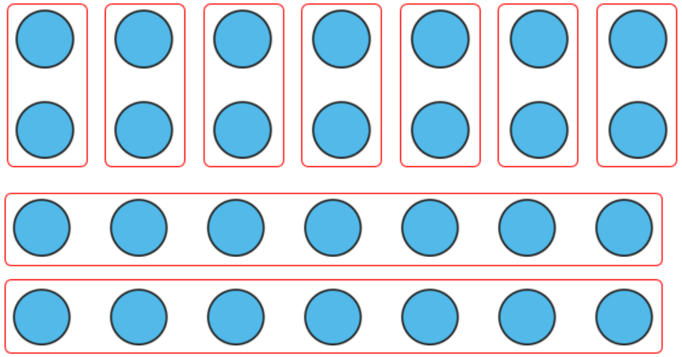
Multiplication and Division

Year 4

Manipulating the Multiplicative Relationship

Vocabulary:

Multiply Divide Commutative Groups of Times Equal to Factors
Product Quotient Dividend Divisor Represents Array



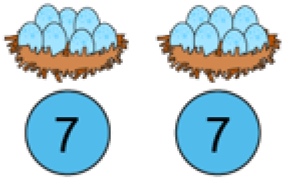
2 × 7 = 7 × 2

Understand that multiplication is commutative and the factors can be

2 groups of 7 is equal to 14.

2, 7 times is equal to 14.

2 groups of 7 is equal to 7, two times.



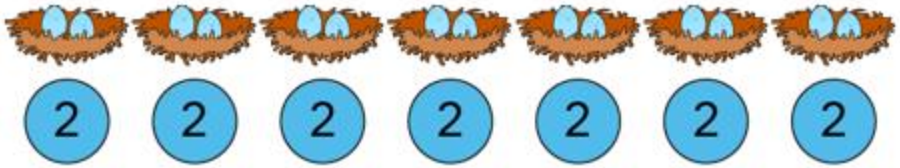
2 × 7 = 14

7 × 2 = 14

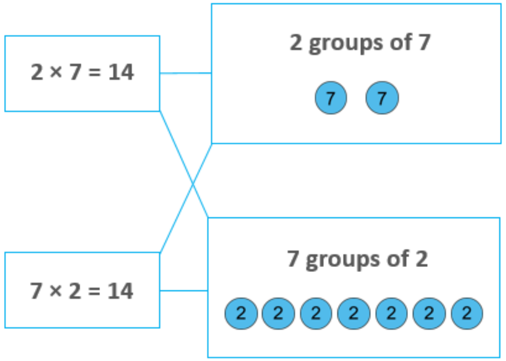
The 2 represents ___.

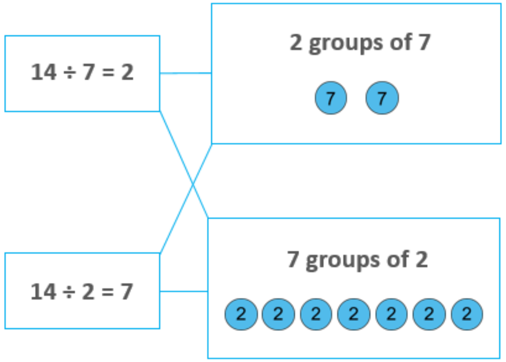
The 7 represents ___.

The 14 represents ___.



Match equations to representations and contexts.





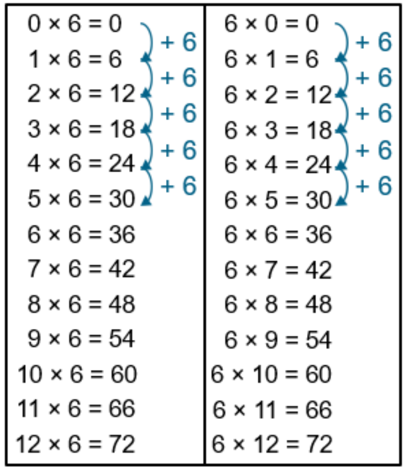
Multiplication and Division

Year 4

The Distributive Property of Multiplication

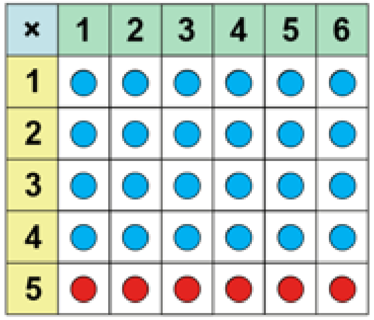
Vocabulary:

Multiplication   Distributive Law   Adjacent   Multiples   Factors   Partitioning
Equations   Expressions   Arrays   Part-whole model   Difference



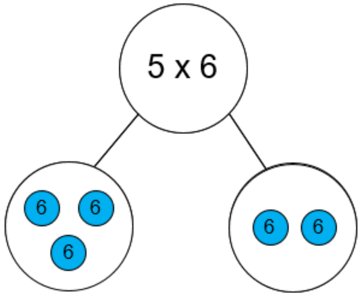
0 × 6 = 0	6 × 0 = 0
1 × 6 = 6	6 × 1 = 6
2 × 6 = 12	6 × 2 = 12
3 × 6 = 18	6 × 3 = 18
4 × 6 = 24	6 × 4 = 24
5 × 6 = 30	6 × 5 = 30
6 × 6 = 36	6 × 6 = 36
7 × 6 = 42	6 × 7 = 42
8 × 6 = 48	6 × 8 = 48
9 × 6 = 54	6 × 9 = 54
10 × 6 = 60	6 × 10 = 60
11 × 6 = 66	6 × 11 = 66
12 × 6 = 72	6 × 12 = 72

Adjacent multiples of ___ have a difference of ___.



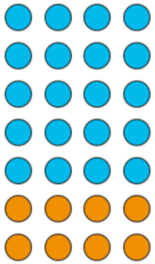
4 × 6 + 6

Five sixes is one more six than four sixes.



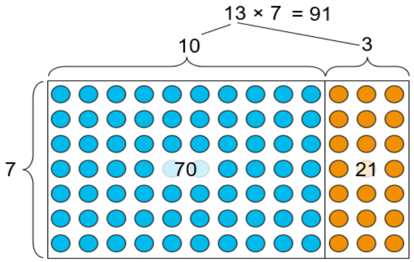
3 x 6 + 2 x 6 = 5 x 6

5 is equal to 3 plus 2, so 5 sixes is equal to 3 sixes plus 2 sixes.

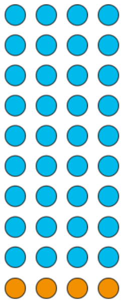


7 = 5 + 2
7 × **4** = 5 × **4** + 2 × **4**
= 20 + 8
= 28

We can partition one of the factors to make calculations easier.

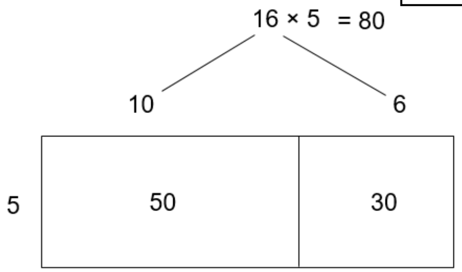


13 × 7 = 10 × 7 + 3 × 7
= 70 + 21
= 91

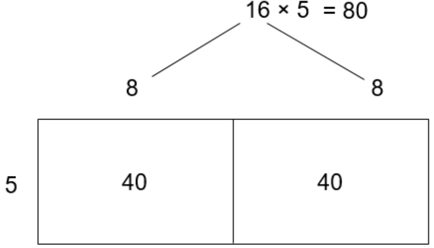


9 = 10 − 1
9 × 4 = **10** × 4 − **1** × 4
= 40 − 4
= 36

We can partition the factors in different ways to make calculations easier.



16 × 5 = 10 × 5 + 6 × 5
= 50 + 30
= 80



16 × 5 = 8 × 5 + 8 × 5
= 40 + 40
= 80

Multiplication and Division

Year 5

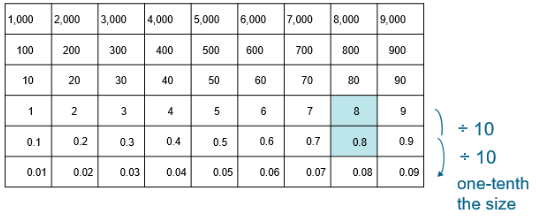
Multiplying and Dividing by 10 and 100 (1)

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

8 ÷ 10 =

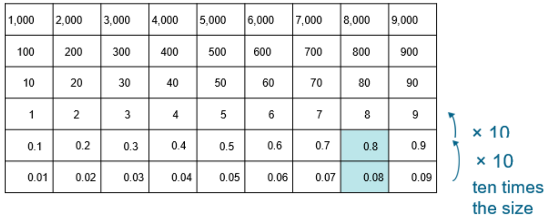
0.8 ÷ 10 =



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

0.08 x 10 =

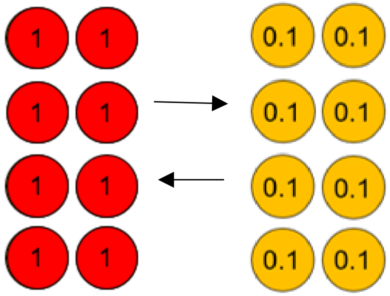
0.8 x 10 =



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

8 ÷ 10 = 0.8

one-tenth of the size



0.8 x 10 = 8

ten times the size

We can multiply and divide a number by 10.

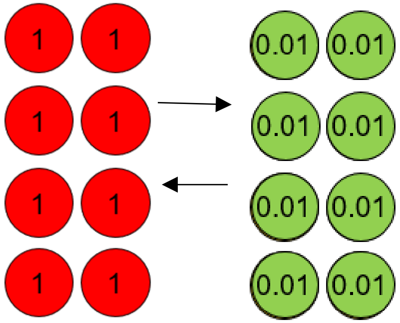
8, made one-tenth the size is 0.8.

8 divided by 10 is 0.8.

First we had 8 ones, now we have 8 tenths.

8 ÷ 100 = 0.08

one-hundredth of the size



0.08 x 100 = 8

one hundred times the size

We can multiply and divide a number by 100. Multiplying by 100 is the same as multiplying/dividing by 10 twice.

8, made 100 times smaller is 0.08.

8 divided by 100 is 0.08.

First we had 8 ones, now we have 8 hundredths

Multiplication and Division

Year 5

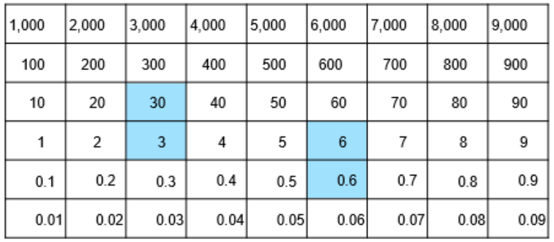
Multiplying and Dividing by 10 and 100 (2)

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

3.6 × 10 = 36

36 ÷ 10 = 3.6



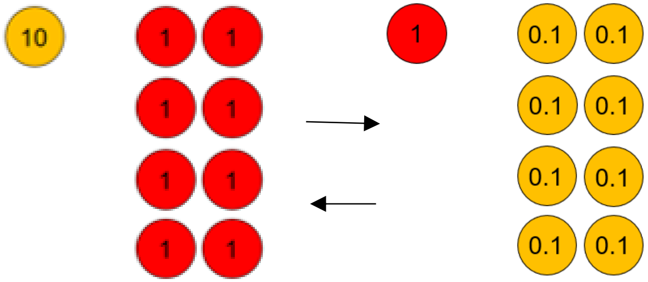
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

18 ÷ 10 = 1.8

one-tenth of the size

1.8 is one-tenth the size of 18

18 divided by 10 is 1.8.



1.8 × 10 = 18

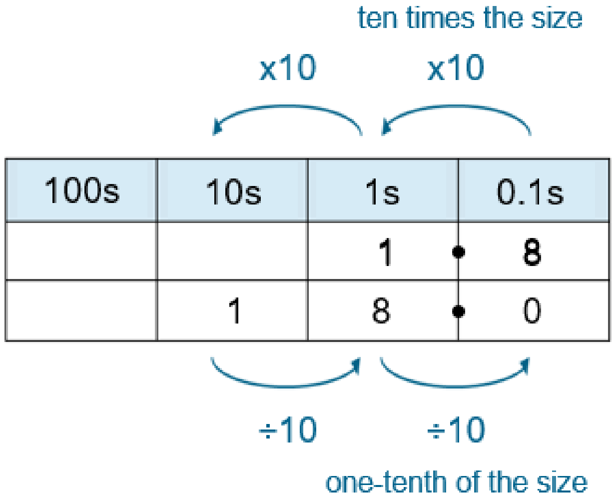
ten times the size

__ divided by 10/100 is equal to__.

__ is one-tenth/hundredth the size of __.

__ multiplied by 10/100 is equal to__.

__ is 10/100 times the size of __.



100s	10s	1s	0.1s
		1 •	8
	1	8 •	0

We can multiply and divide numbers with digits greater than 0 by 10 or 100.

Generalisation

To multiply by 10, move each digit one place to the left.

To multiply by 100, move each digit two places to the left.

To divide by 10, move each digit one place to the right.

Multiplication and Division

Year 5

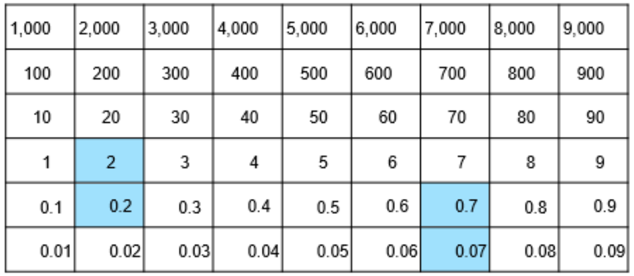
Multiplying and Dividing by 10 and 100 (3).

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size One-hundredth the size Gattegno chart Factor Product Multiple Groups of Inverse Ones Tens Hundreds Tenths Hundredths

0.27 × 10 = 2.7

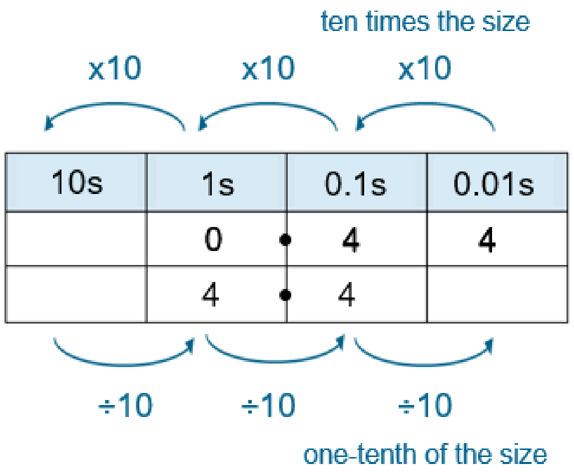
2.7 ÷ 10 = 0.27



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

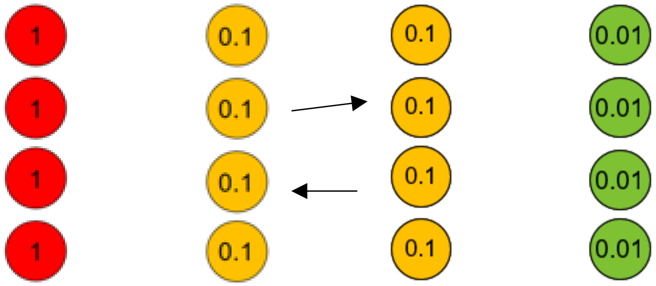
0.27 is one-tenth the size of 2.7

2.7 divided by 10 is 0.27.



4.4 ÷ 10 = 0.44

one-tenth of the size



0.44 × 10 = 4.4

ten times the size

__ divided by 10/100 is equal to__.

__ is one-tenth/hundredth the size of __.

__ multiplied by 10/100 is equal to__.

__ is 10/100 times the size of __.

We can multiply and divide numbers with digits greater than 0 by 10 or 100.

Generalisation

To multiply by 10, move each digit one place to the left.

To multiply by 100, move each digit two places to the left.

To divide by 10, move each digit one place to the right.

Multiplication and Division

Year 5

Find Factors and Multiples

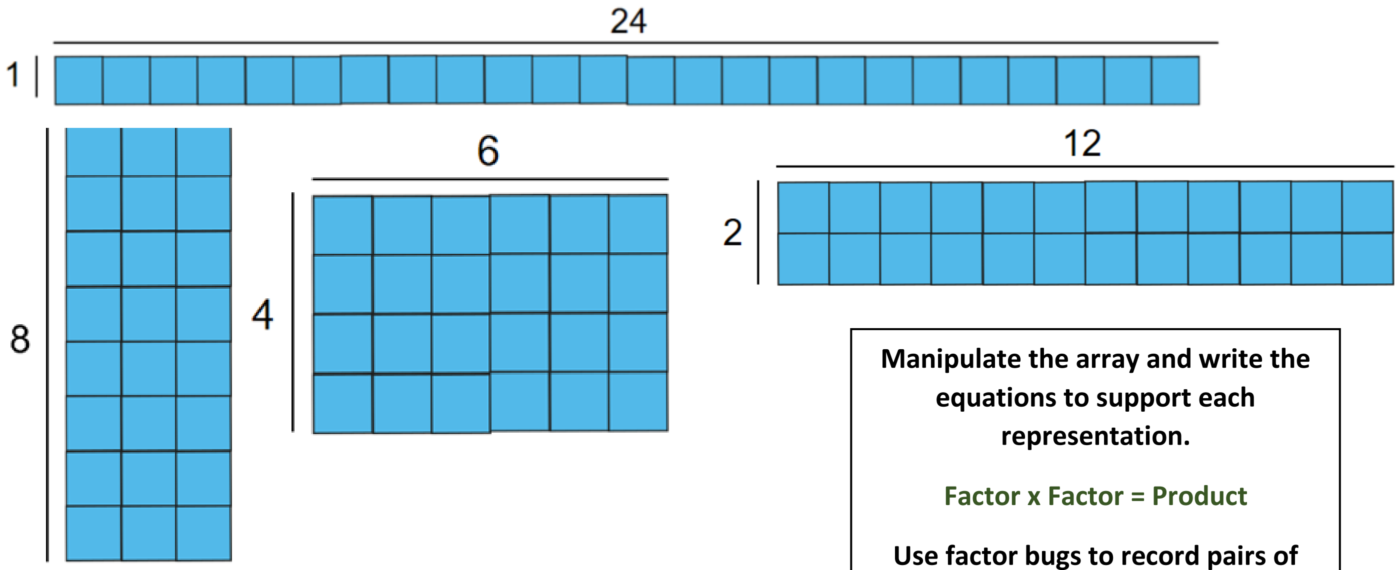
Vocabulary:

Factor Multiple Composite Square Prime Common Factor Prime Factor

Factor Bug Array Positive Integer Working Systematically

Factor x Factor = Product

Dividend ÷ Divisor = Quotient



8 x 3 = 24

4 x 6 = 24

2 x 12 = 24

1 x 24 = 24

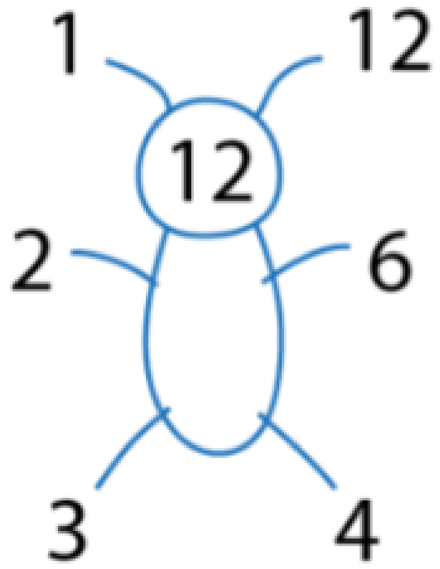
There are ___ tiles. There are ___ rows and ___ columns. So ___ and ___ are factors of ___.

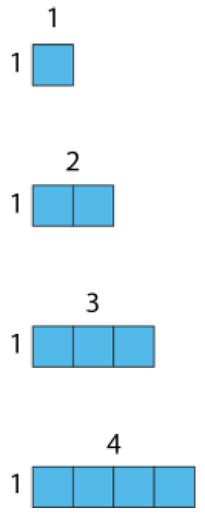
Generalise: Numbers that have more than two factors are composite numbers.

Manipulate the array and write the equations to support each representation.

Factor x Factor = Product

Use factor bugs to record pairs of factors.





1 × 1 = 1

2 × 1 = 2

3 × 1 = 3

4 × 1 = 4

Generalise:

When one is a factor, the product is equal to the other factor.

All positive integers have a factor of 1.

Every positive integer is a factor of itself.

The smallest factor of a positive integer is always 1. The largest factor of a positive integer is always itself.

Multiplication and Division

Year 5

Find Factors and Multiples

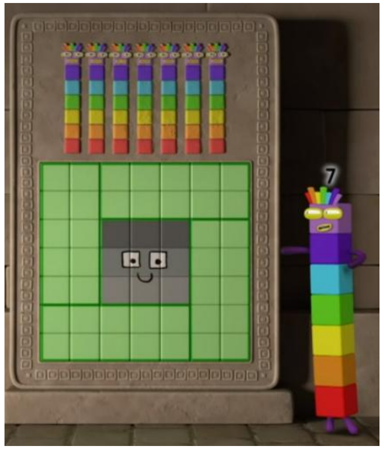
Vocabulary:

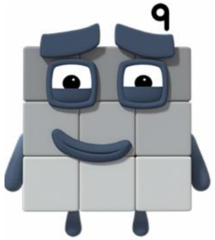
Factor Multiple Composite Square Prime Common Factor Prime Factor

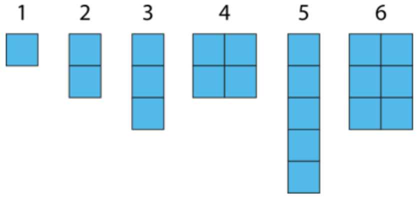
Factor Bug Array Positive Integer Working Systematically

Factor x Factor = Product

Dividend ÷ Divisor = Quotient







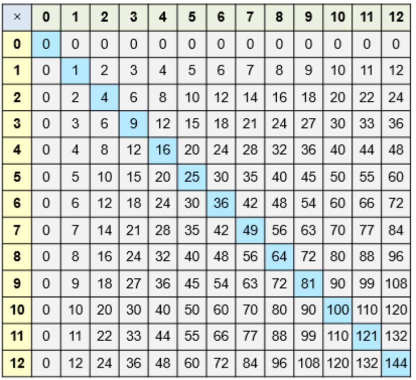
Make connections with factors and times tables. Make connections with factors of factors

___ is a factor of ___ because it is in the ___ times table.

Nine is a factor of all of these numbers.

Three is a factor of nine which means it is also a factor of all of these numbers.

Extend this to square numbers, and prime numbers recognising the number of factors.

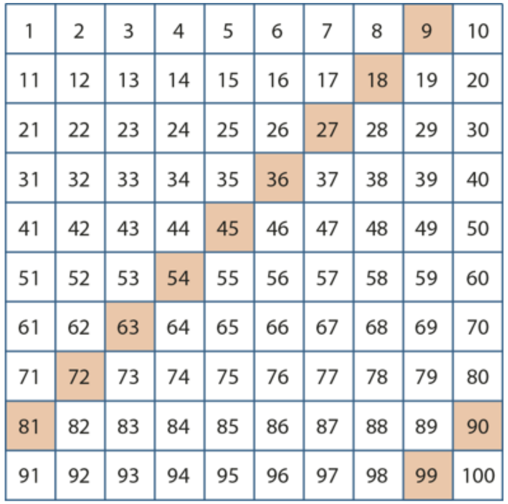


×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Is 9 a factor of 54?

54 ÷ 9 = 6

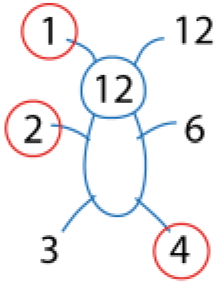
9 and 6 are factors of 54.

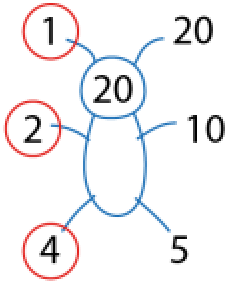


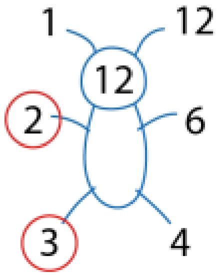
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

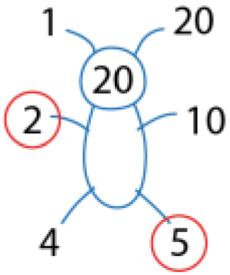
Use factor bugs to find

common factors and prime factors.









Multiplication and Division

Year 5

Find Factors and Multiples

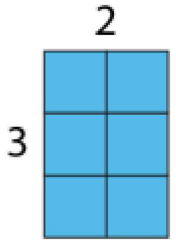
Vocabulary:

Factor Multiple Composite Square Prime Common Factor Prime Factor

Factor Bug Array Positive Integer Working Systematically

Factor x Factor = Product

Dividend ÷ Divisor = Quotient



Introduce Multiples

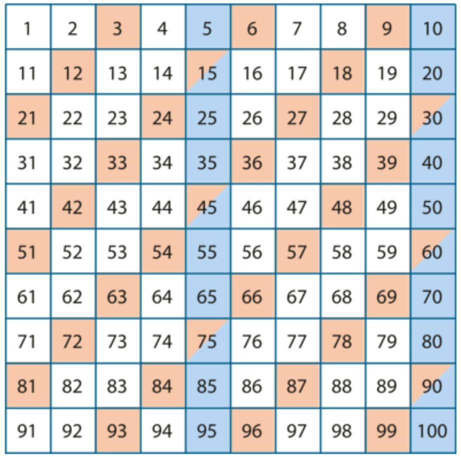
___ is a factor of ___ because ___ x ____ = ____.

___ is a multiple of ___ because ___ x ___ = ____

___ is a factor of ___ because ___ ÷ ____ = ____.

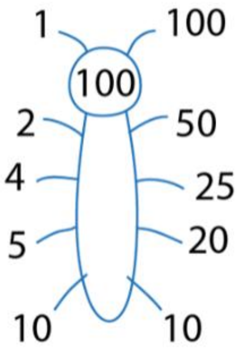
___ is a multiple of ___ because ___ ÷ ___ = ____

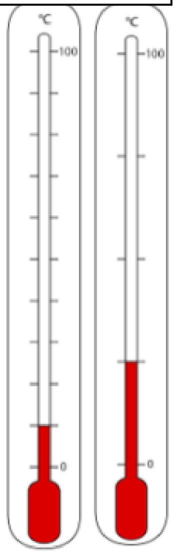
Identify Common Multiples using a 100 square.

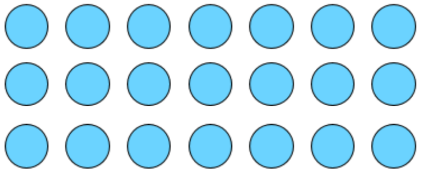


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Factors of 100 can be applied to contexts







7 × 3 = 21

Make statements about factors and multiples whilst increasing the amount of each counter in the array.

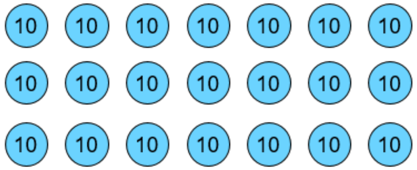
___ represents the number of counters in each row.

___ represents the total value of the counters in each column.

_____ represents the total value of the counters.

3, 7, 10, 21 and 70 are factors of 210.

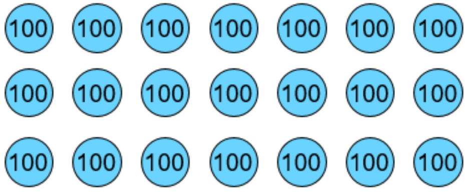
210 is a multiple of 3, 7, 10, 21 and 70.



7 × 30 = 210

70 × 3 = 210

10 x 21 = 210



7 × 300 = 2,100

700 × 3 = 2,100

100 x 21 = 2,100

Multiplication and Division

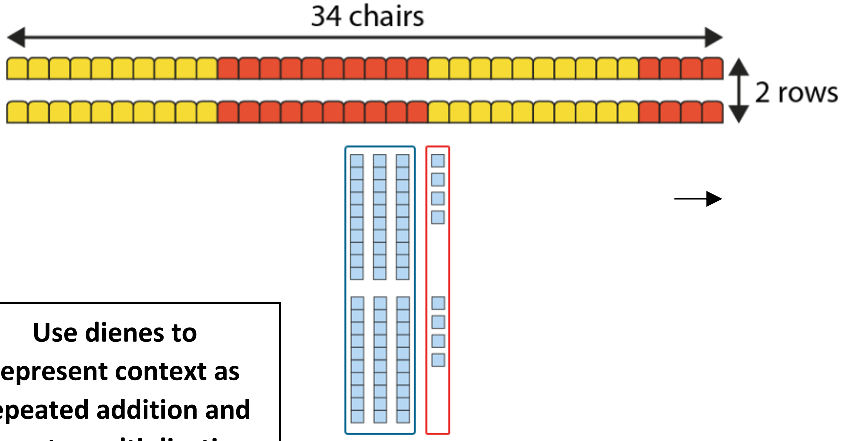
Year 5

Multiply using a Formal Written Method (1)

Vocabulary:

Ones Tens Hundreds Thousands Represents Partition Recombine
Multiply Unitising Partial Product Aligned Calculation Expanded layout
Compact layout Equation Regroup Algorithm

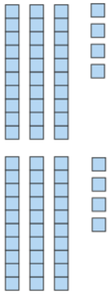
Factor x Factor = Product



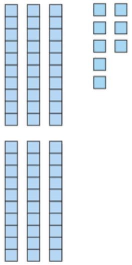
34 = 30 + 4
34 × 2 = 30 × 2 + 4 × 2

Use dienes to represent context as repeated addition and move to multiplication.

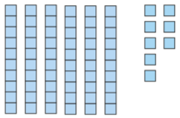
Move between representations of dienes and expanded written multiplication.



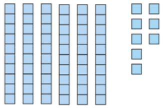
	10s	1s
	3	4
×		2



	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones



	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones
	6	0	2 × 3 tens = 6 tens

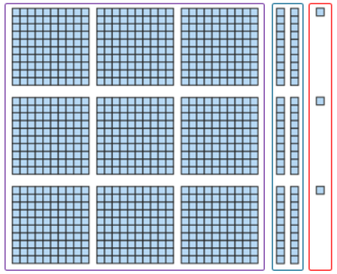


	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones
	6	0	2 × 3 tens = 6 tens
	6	8	

	10s	1s
	3	4
×		2
	6	8

Move between representations of expanded layout and compact layout.

321 × 3 = 963



	100s	10s	1s	
	3	2	1	
×			3	
			3	3 × 1 ones = 3 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	3	

	3	2	1
×			3
	9	6	3

Represent 3 digit by 1 digit multiplication without exchanges using dienes, moving from expanded layout to compact layout.

Multiplication and Division

Year 5

Multiply using a Formal Written Method (2)

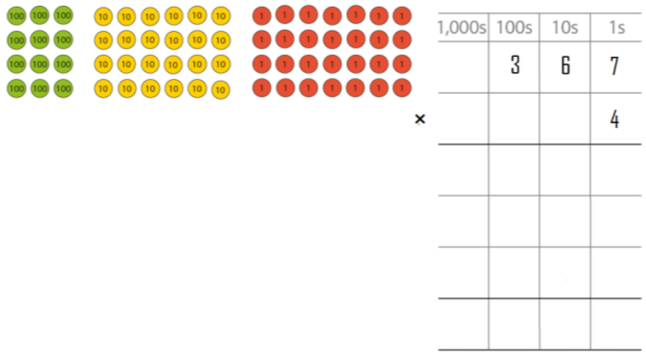
Vocabulary:

Ones Tens Hundreds Thousands Represents Partition Recombine
Multiply Unitising Partial Product Aligned Calculation Expanded layout
Compact layout Equation Regroup Algorithm

Factor x Factor = Product

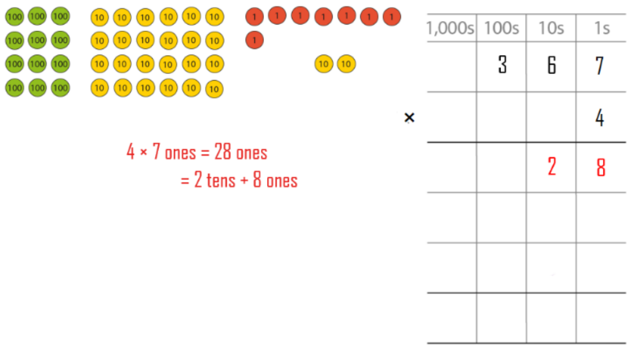
Represent 3 digit by 1 digit multiplication with exchanges using place value counters, moving from expanded layout to compact layout.

367 × 4 = ☐



	1,000s	100s	10s	1s
		3	6	7
×				4

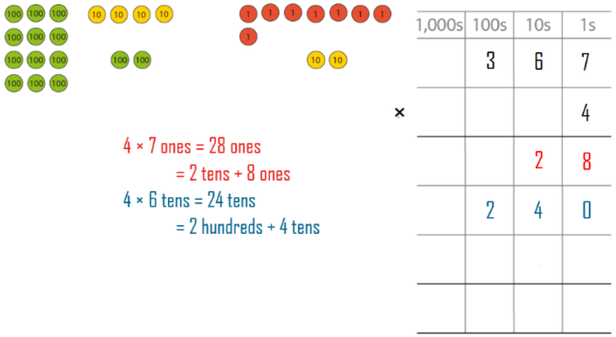
367 × 4 = ☐



4 × 7 ones = 28 ones
= 2 tens + 8 ones

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8

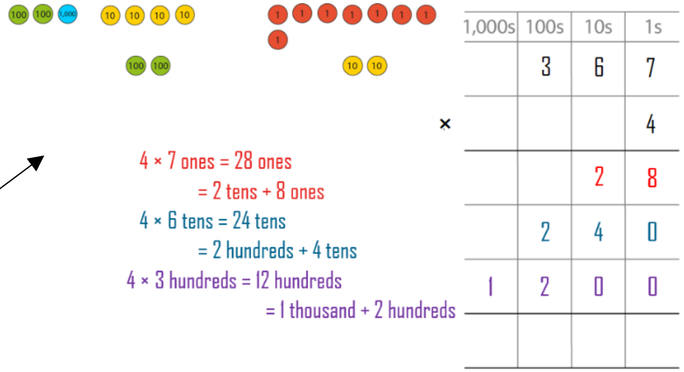
367 × 4 = ☐



4 × 7 ones = 28 ones
= 2 tens + 8 ones
4 × 6 tens = 24 tens
= 2 hundreds + 4 tens

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8
		2	4	0

367 × 4 = ☐



4 × 7 ones = 28 ones
= 2 tens + 8 ones
4 × 6 tens = 24 tens
= 2 hundreds + 4 tens
4 × 3 hundreds = 12 hundreds
= 1 thousand + 2 hundreds

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8
		2	4	0
	1	2	0	0

367 × 4 = 1,468

Move from expanded layout to compact layout.

		3	6	7
×				4
	1	4	6	8
		2	2	

If there are 10 or more ones, we must regroup ones into tens and ones.

If there are 10 or more tens, we must regroup into hundreds and tens.

If there are 10 or more hundreds, we must regroup into thousands and hundreds.

Multiplication and Division

Year 5

Divide using a Formal Written Method (1)

Vocabulary:

Partitive (sharing) Quotitive (grouping) Ones Tens Hundreds Thousands Represents Divide Unitising Dividend Divisor Quotient Partial Quotient Aligned Calculation Equation Exchange Algorithm 'Sharees' Divisible Remainder Short Division

84	÷	4	=	21	$\begin{array}{r} 2\ \ 1 \\ 4\overline{)\,8\ \ 4} \end{array}$
dividend	÷	divisor	=	quotient	$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\,\text{dividend}} \end{array}$

Use sticks to represent partitive (sharing) context where the dividend is divisible (to give a whole number). Skip count in multiples of the divisor.

84 sticks shared equally between 4 children. How many sticks each?

84 ÷ 4 = ☐

Step 1 – write the divisor and dividend:

10s 1s

$4\overline{)\,8\quad 4}$

Step 2 – share the 10s:

10s 1s

$\begin{array}{r} 2 \\ 4\overline{)\,8\quad 4} \end{array}$

8 tens ÷ 4 = 2 tens

8 tens divided by 4 is equal to 2 tens.

Remove a group of 4 each time before sharing.

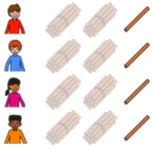
One four is one each. That's four.

Two fours is two each. That's eight.

Eight divided between 4 is equal to two each.

Each child gets two sticks.

Step 3 – share the 1s:



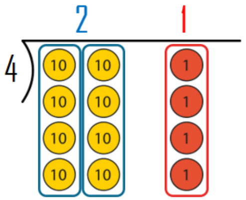
10s 1s

$\begin{array}{r} 2\quad 1 \\ 4\overline{)\,8\quad 4} \end{array}$

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

10s 1s



8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

Add the partial quotients to find the quotient.

2 tens + 1 one = 21

Multiplication and Division

Year 5

Divide using a Formal Written Method (2)

Vocabulary:

Partitive (sharing) Quotitive (grouping) Ones Tens Hundreds Thousands
Represents Divide Unitising Dividend Divisor Quotient Partial Quotient
Aligned Calculation Equation Exchange Algorithm 'Sharees' Divisible Remainder
Short Division

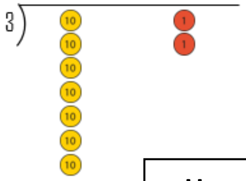
84	÷	4	=	21	$\begin{array}{l}\ \ \ 2\ \ 1\\ 4\overline{)\,8\ \ 4}\end{array}$
dividend	÷	divisor	=	quotient	$\begin{array}{r}\text{quotient}\\ \text{divisor}\overline{)\,\text{dividend}}\end{array}$

72 sticks shared equally between 3 children. How many sticks each?

72 ÷ 3 = ☐

Step 1 – write the divisor and the dividend:

$$3\overline{)\,7\ \ \ 2}$$



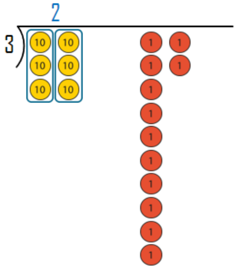
Step 2 – share the 10s:

$$\begin{array}{l}\ \ \ \ 2\\ 3\overline{)\,7\ \ \ 2}\end{array}$$

7 tens ÷ 3 = 2 tens r 1 ten

Step 3 – exchange:

$$\begin{array}{l}\ \ \ \ 2\\ 3\overline{)\,7\ \ {}^{1}2}\end{array}$$

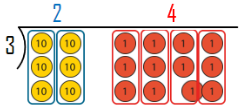


7 tens ÷ 3 = 2 tens r 1 ten

72 ÷ 3 = 24

Step 4 – share the 1s:

$$\begin{array}{l}\ \ \ \ 2\ \ \ 4\\ 3\overline{)\,7\ \ {}^{1}2}\end{array}$$



7 tens ÷ 3 = 2 tens r 1 ten

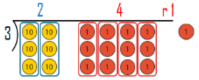
12 ones ÷ 3 = 4 ones

Use sticks and place value counters to represent partitive (sharing) context where the dividend is divisible (to give a whole number) though requires an exchange from the tens. Skip count in multiples of the divisor.

If dividing the tens gives a remainder of one or more ten, we must exchange the remaining tens for ones.

73 ÷ 3 = 24 r 1

$$\begin{array}{l}\ \ \ \ 2\ \ 4\,r1\\ 3\overline{)\,7\ \ {}^{1}3}\end{array}$$



7 tens ÷ 3 = 2 tens r 1 ten

13 ones ÷ 3 = 4 ones r 1 one

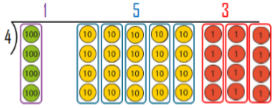
Apply the same representations though this time include a remainder.

Then extend to division of 3 digits by one digit and where there can be no hundreds cannot be shared.

If dividing the hundreds gives a remainder of one or more hundred, we must exchange the remaining hundreds for tens.

612 ÷ 4 = 153

$$\begin{array}{l}\ \ \ \ 1\ \ 5\ \ 3\\ 4\overline{)\,6\ \ {}^{2}1\ \ {}^{1}2}\end{array}$$



6 hundreds ÷ 4 = 1 hundred r 2 hundreds
2 hundreds = 20 tens
21 tens ÷ 4 = 5 tens r 1 ten
1 ten = 10 ones
12 ones ÷ 4 = 3 ones

Addition, Subtraction, Multiplication and Division

Year 6

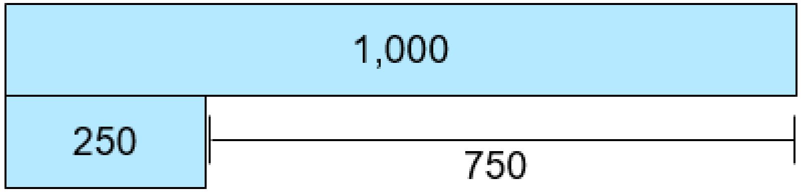
Quantify additive and multiplicative relationships

Vocabulary:

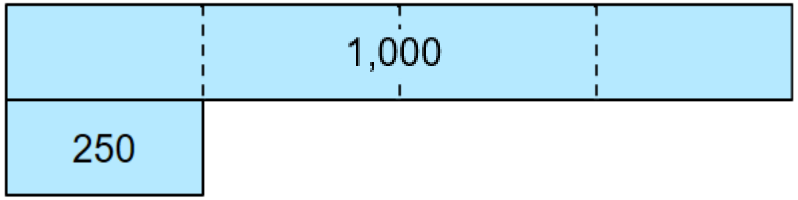
Additive Multiplicative Relationship Represents Compose Combine Total
More than Less than Plus + Minus - Equal to = Addition Subtraction Divide ÷
Multiply x One-_____ of Equation Expression Bar Model Whole Part
Difference Multiplier Unknown Sequence

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

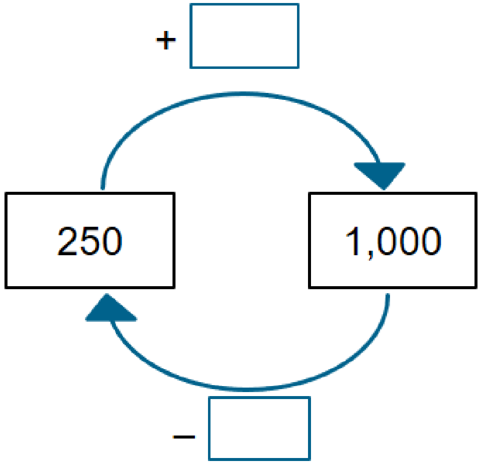


250 + 750 = 1,000 1,000 – 750 = 250



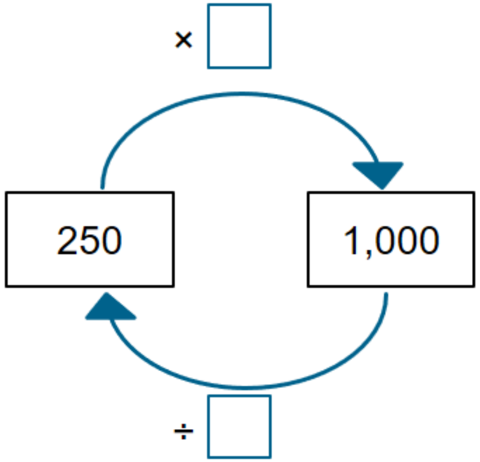
250 x 4 = 1,000 1000 ÷ 4 = 250

The relationship between two numbers can be expressed both additively and multiplicatively.



1000 is ___ more than 250.

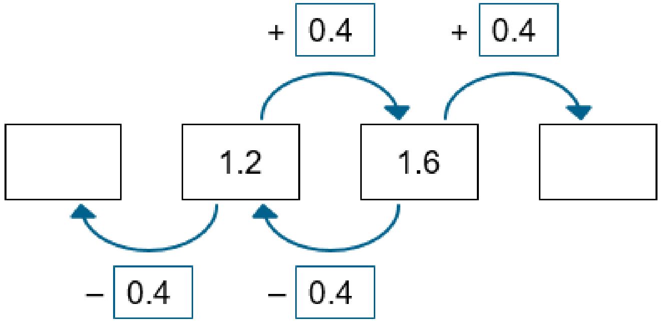
250 is ____ less than 1000.



1000 is ___ times the size of 250.

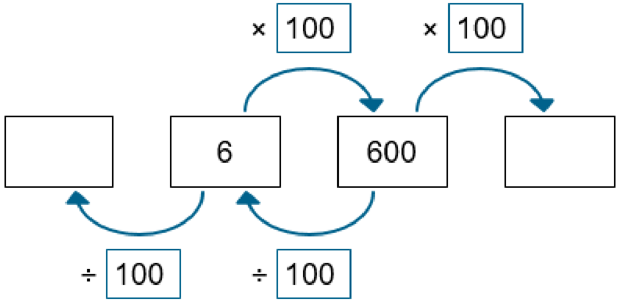
250 is one-_____ of 1000.

To find one-quarter of a number, we divide by 4.



Finding the difference can help calculate the unknown terms in a sequence.

Finding the known multiplier can help calculate the unknown terms in a sequence.



Addition, Subtraction, Multiplication and Division

Year 6

Quantify additive and multiplicative relationships

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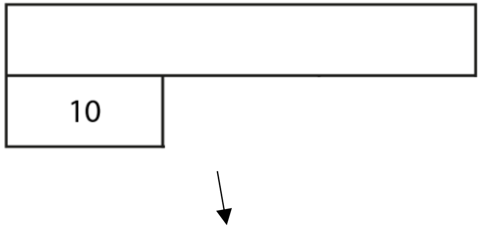
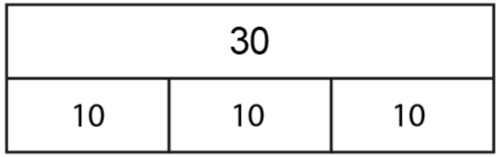
$\frac{1}{3}$ of ?= 10



$\frac{1}{3}$ of ?= 10



Calculate the unknown whole by recognising how many parts the whole has been divided into.

$\frac{1}{3}$ of 30 = 10

Addition and Subtraction

Year 6

Derive Related Calculations

Vocabulary:

Additive Multiplicative Relationship Represents Equation Unknown Re-arrange Inverse Place Value Properties Commutative Associative Distributive Compensation

Addend + Addend = Sum Factor x Factor = Product (Multiplicand x Multiplier = Product)

Minuend – Subtrahend = Difference Dividend ÷ Divisor = Quotient

252 = 3 × 84

2,520 = 30 × ☐

252 = 3 × 84

☐ = 3 × 85

252 = 3 × 84

252 = 3 × 60 + 3 × ☐

625 – 148 = 477

6,250 – 1,480 = ☐

625 – 148 = 477

625 – 70 – ☐ = 477

625 – 148 = 477

625 – 248 = ☐

14.8 + 7.6 = 22.4

1,480 + ☐ = 2,240

14.8 + 7.6 = 22.4

☐ – 7.6 = 14.8

14.8 + 7.6 = 22.4

12.8 + ☐ = 22.4

4,800 ÷ 25 = 192

25 × 192 = ☐

4,800 ÷ 25 = 192

4,800 ÷ 250 = ☐

4,800 ÷ 25 = 192

4,800 ÷ 5 ÷ 5 = ☐

Manipulate an equation to solve another. Pupils could:

- **rearrange the terms;**
- **rewrite using inverse operations;**
- **apply place value;**
- **use the properties of division that correspond to the commutative, associative or distributive property of multiplication;**
- **use the compensation property.**

Additive examples

Multiplicative examples

Addition and Subtraction

Year 6

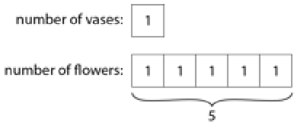
Solve Problems involving Ratio Relationship

Vocabulary:

Additive Multiplicative Relationship Represents Equation Unknown Scale-factor Ratio Ratio Table ___ times the size one-___ the size of Vertical Horizontal

Factor x Factor = Product (Multiplicand x Multiplier = Product)

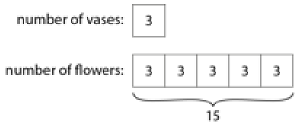
Dividend ÷ Divisor = Quotient



$1 \times \mathbf{5} = 5$

$5 \div \mathbf{5} = 1$

$5 \times \frac{1}{5} = 1$



$3 \times \mathbf{5} = 15$

$15 \div \mathbf{5} = 3$

$15 \times \frac{1}{5} = 3$

Ratio table to compare sets of information.

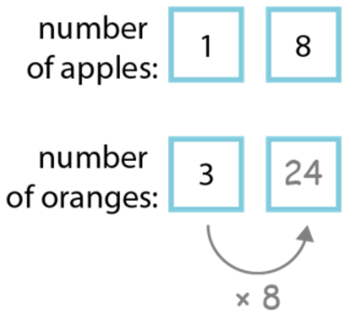
For every ___, there are ___.

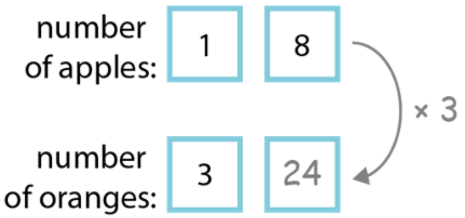
For every 1 litre of petrol, you can drive 7 miles.

For every 7 miles you will drive, you need 1 litre of petrol.

Extend sequences using knowledge of patterns based on ratio table.

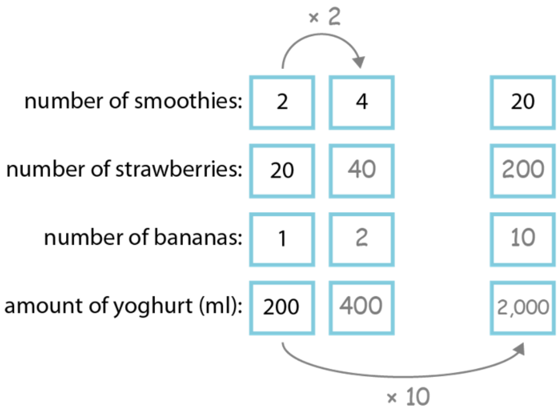
Litres of petrol	1	2	3	4	5	6	7	8	9	10
Miles driven	7	14	21	28	35	42	49	56	63	70





Explore vertical and horizontal relationship between numbers.

For every ___, there are ___.

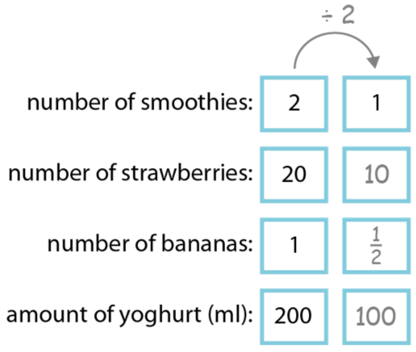


Identify the scale-factor in order to find unknown values.

___ is ___ times the size of ___.

Therefore I must multiply/divide by ___.

___ is one-__ the size of ___.



Addition and Subtraction

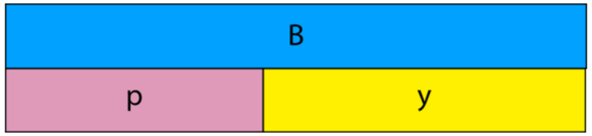
Year 6

Solve Problems with Two Unknowns

Vocabulary:

Additive Multiplicative Relationship Represents Equation Two Unknowns
Scale-factor Ratio ___ times the size one-___ the size of Total Bar Model
Structure

$$B = \boxed{r} + \boxed{b}$$



$$B = \boxed{p} + \boxed{y}$$

Use Cuisenaire to find 2 bars of total length that are equal to another.

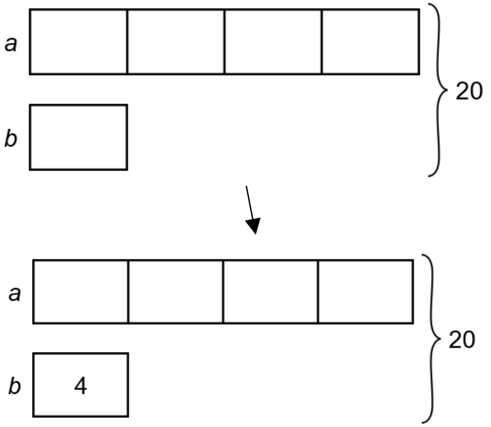
There is more than one solution to the problem.

There can be infinite solutions to a problem.

$$5 \times \boxed{} = 10 \times \boxed{}$$

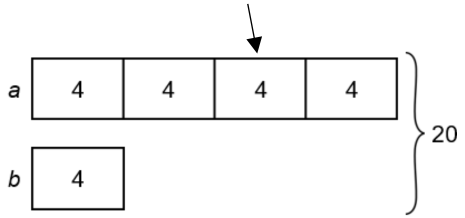
Solve multiplicative problems with two unknowns when the total is known.

v



one part = 20 ÷ 5 = 4

b = 4



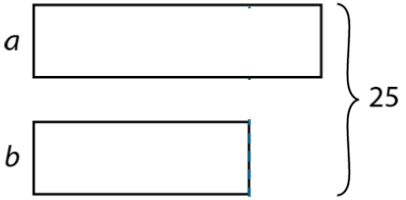
one part = 20 ÷ 5 = 4

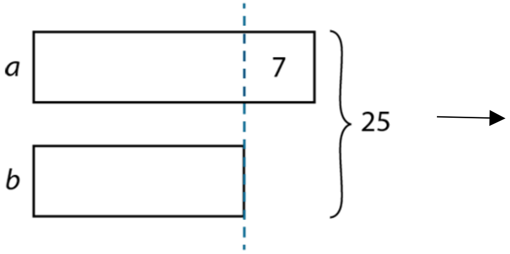
b = 4

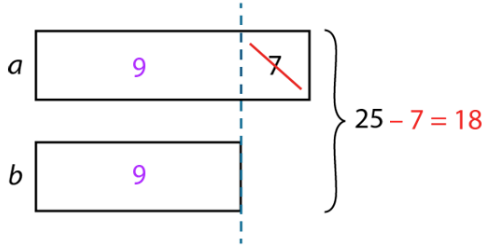
a = 4 × 4 = 16

The two numbers are 16 and 4.

Solve additive problems with two unknowns when the total is known.







b = 18 ÷ 2 = 9

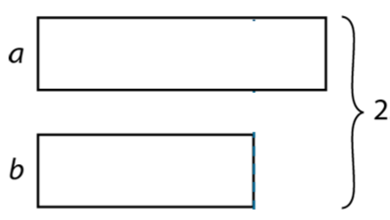
a = 9 + 7 = 16

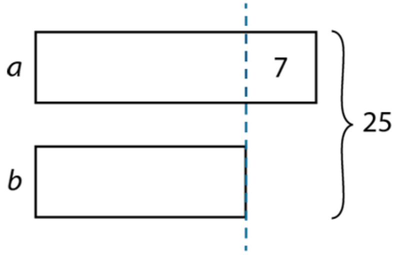
The two numbers are 9 and 16.

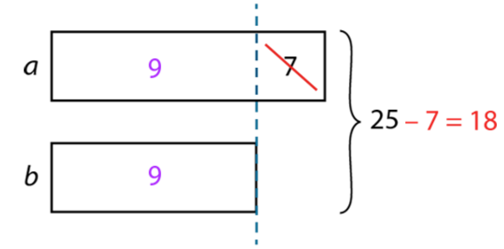
v

$$5 \times \square = 10 \times \square$$

Solve additive problems with two unknowns when the total is known.



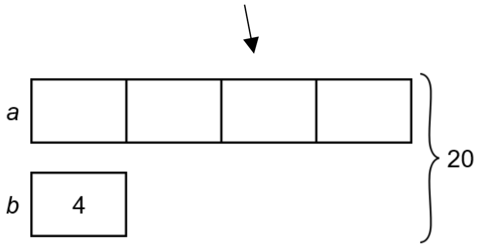




b = 18 ÷ 2 = 9

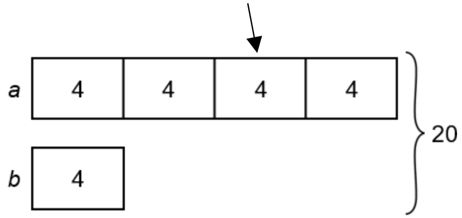
a = 9 + 7 = 16

The two numbers are 9 and 16.



one part = 20 ÷ 5 = 4

b = 4



one part = 20 ÷ 5 = 4

b = 4

a = 4 × 4 = 16

The two numbers are 16 and 4.